Large number estimation as a vehicle to promote mathematical modelling

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This article presents a teaching experiment study in which year 2 students compare city and town based on population estimates. The activities are presented to them as Fermi problems and require an analysis of the reality that allows the identification of sub-problems that the students can face. The students' products are analysed from the perspective of mathematical modelling and it is observed that the students develop their own mathematical methods to make the required estimates. Learning opportunities are identified when students require concepts and procedures that have not yet been worked on. The results show that estimation acts as a facilitator for the generation of mathematical models that allow students to connect their real-world knowledge with new mathematical learnings. In the conclusions, the implications of the use of Fermi problem sequences to initiate mathematical modelling and connecting mathematical and extra-mathematical knowledge about social problems in the first years of Primary Education are discussed.

Keywords: Mathematics; mathematical modelling; estimation; Fermi problems, Primary school

Introduction

This study presents a teaching experiment aimed at exploring the potential of large number estimation as a vehicle to introduce mathematical modeling to second-grade students (7 to 8 years old). It is known that children think mathematically long before they start school (Butterworth 2005) and mathematical thinking is a strong predictor for later academic success at school (Claessens et al. 2009; Duncan et al. 2007). For this reason, it might be interesting to deepen mathematical modeling development at early ages as a way of developing mathematical competence. Few studies on mathematical modeling with students under 10 years of age have been carried out in the last few decades (English 2006, 2012; Greer 1993; Lehrer and Schauble 2000; Verschaffel and De Corte 1997). Consequently, some aspects of mathematical modeling at early ages have not been properly explored, such as the real, concrete contexts that enable students to incorporate their own knowledge about the real world into a modeling activity (Stohlmann and Albarracín 2016). This research fills this gap by studying a mathematical modeling project—called City versus Town—in which students learned to understand the differences between towns and cities.

The project took place in a state school in a city of 200,000 inhabitants (Sabadell, Spain), with a high population density. It was intended that the students compare it with a town they would visit on an excursion a few weeks after the project (Sant Sadurní d'Anoia, Spain, 13,000 inhabitants). In this school, different ways of working in the classroom are used. Those include textbook-oriented classes, the use of digital materials, and multidisciplinary projects in which students try to understand a social issue, an artistic style, a cultural event, an environmental problem, or some other reality. Although in these projects mathematics does not usually play a primary role, the City versus Town project was totally geared towards promoting the development of mathematical models to describe reality while working on the estimation of large numbers. In this respect, several activities were proposed to students to guide them to find ways of comparing towns and cities from a mathematical perspective.

In particular, the aim of this study was to identify the mathematical models and learning opportunities put into practice or promoted when working on the estimation of large numbers with second-grade primary school students. To this end, these pupils estimated and compared the populations of a city and a town.

Mathematical Modeling

There are several conceptions of mathematical modeling in the literature that highlight different educational issues but share a common vision of mathematical modeling as a problem-solving process linking the real world and mathematics (Abassian et al. 2020; Blomhøj 2009; Kaiser and Sriraman 2006). Mathematical modeling involves mathematizing real-world situations and preparing mathematical models to describe the phenomena studied. Lesh and Harel (2003) define mathematical models as conceptual systems developed to describe other systems. These authors understand models as a set of concepts intended to describe or explain the mathematical objects and patterns relevant to the phenomenon studied, together with the corresponding procedures needed to generate constructions, manipulations, and predictions that can be used to achieve clearly identified objectives.

During modeling activities, students try to solve problems by progressing through different stages, moving from reality to the mathematical domain, and then going back to reassess the phenomenon under study. Therefore, the whole process is repeated in different cycles in which students improve the models and solutions found for the problem they are working on, adapting them to the requirements of the problem statement (Blum 2015; Blum and Borromeo Ferri 2009). To move successfully through these stages, students must activate a series of competencies that include aspects related to metacognition, motivation, and their own beliefs about the nature of mathematics (Maaß 2006).

In this paper a theoretical perspective called Models & Modeling (M&M) is considered, where modeling includes multiple cycles of interpretations, descriptions, conjectures, explanations and justifications that are iteratively redefined and reconstructed through student interaction (Doerr and English 2003). Modeling involves several processes such as quantifying, dimensioning, coordinating, categorizing, and systematizing different kinds of relevant objects, relationships, actions, patterns, and regularities (Mousoulides et al. 2007). Lesh et al. (2000) established the fundamental principles of the M&M perspective, disassociating the modeling activities belonging to the M&M perspective from applied mathematics. The problems, contextualized in the real world, and with characteristics and demands that turn them into modeling activities, are called 'model eliciting activities' (MEAs) and include both the process (modeling) and the product (model). Thus, the approach to the problem must enable students to establish appropriate criteria to help them decide which solution is appropriate among a set of different alternatives. Students are usually asked to work in small groups (Zawojewski et al. 2003) and are confronted with a problematic situation that is significant and relevant to them, and for which they need to create, expand, and improve their own mathematical constructions. Once the model has been obtained, students must determine whether it is only applicable to the particular situation presented in the problem or whether, on the contrary, the model can be shared, moved, easily modified and reused for other types of problems or situations. When using MEAs with preschool to second-year primary school students, Lesh et al. (2013) noted several key points for implementation, such as the importance of focusing on conceptual knowledge as well as procedural knowledge, choosing a small number of "big ideas" that can be useful outside school and laying the emphasis on research-based learning progressions.

Estimation and Fermi Problems

An estimate is made when answering questions such as: How long would I take to walk to school? Would all these textbooks fit in a small satchel or should I use a larger one? Estimation involves making a judgment on the value of the result of a numerical operation or the measurement of a quantity, depending on the individual circumstances of the estimator. Generally, one can differentiate between computational estimation, measurement estimation, and numerosity as different methods of estimation based on the skillset required to achieve the estimated value (Hogan and Brezinski 2003). However, if they have the same goal, the three taken together can be understood as a mathematical process that gives a rough solution to a problem of measurement or counting (Siegel et al. 1982). One characteristic of estimation is that it is often done mentally, quickly, and using as simple numbers as possible, which means that while the value assigned to the quantity does not have to be exact it should be sufficiently precise, which depends on the context and needs of the person making the estimate (Reys 1984). In primary education, measurement estimation is a tool included in the mathematics syllabus as a way of introducing measurement content, but it is also used to consolidate this content (Sarama and Clements 2009) by creating a well-defined set of mental units of reference which serve to support their estimates (Bright 1976).

One kind of estimation problems used in several scientific disciplines to introduce modeling and develop problem-solving competencies consists of the so-called Fermi problems (Ärlebäck and Albarracín 2019). Fermi problems are defined by Ärlebäck (2009) as realistic, open, non-standard problems where the students are required to make assumptions about the problem situation and estimate relevant quantities before engaging in simple calculations. Although they originated in the teaching of physics (Robinson 2008), in recent decades Fermi problems have attracted greater interest and been used in other subjects and disciplines, including mathematics teaching and learning. Fermi problems offer little or no specific information as regards guiding the problem-solving process, which means they can be connected to different curricular content (Efthimiou and Llewellyn 2007). Examples of Fermi problems previously used in the mathematics classroom are: How much paper does your school use in one month? Or, if there is a three-kilometer tailback on a highway, how many vehicles are caught in this traffic jam? (2009). Work methods combining Fermi problems and mathematical modeling have been identified among college students (Czocher 2016), secondary school students (Albarracín and Gorgorió 2014; Ärlebäck 2009), and also 10- to 12-year-old primary school students (Albarracín and Gorgorió 2019; Peter-Koop 2009). Primary school students have proved to be capable of identifying the key variables of the studied phenomenon, establishing their relationships, determining the necessary values through estimation or other methods, and calculating the values required in the problem statement.

Fermi problems can be solved by the Fermi estimates method, decomposing the original problem into simpler sub-problems and making reasonable estimates for each sub-problem to reach a solution to the original question (Carlson 1997). Research suggests that this method can be understood by high school and tertiary students and successfully taught to them (Raviv et al. 2016; Barahmeh et al. 2017) but there is no published research on the use of the Fermi estimates method at earlier ages. This way of using estimation is considered a facilitator in the solving of real context activities (Chesnutt et al. 2018; Resnick et al. 2017). It has been also suggested that working on Fermi problems involving socially relevant contexts provides a way of enhancing critical thinking and can be used to understand the world around us (Sriraman and Knott 2009).

Fermi problems can be presented to students as isolated activities or as part of a sequence of activities. From the M&M perspective, the work has to start in the classroom by proposing activities that enable students to develop their own mathematical model—Model-Eliciting Activities (MEAs), and then proposing Model Exploration Activities (MXAs) that enable them to adapt this model to other situations (Ärlebäck and Doerr 2015). Up to now, sequences that present Fermi problems in different contexts have been used with high school students to promote the development of models based on the concept of population density (Albarracín and Gorgorió 2018). In the case of students in the first years of primary education, this strategy looks very promising. Just recently, Pla-Castells and Ferrando (2019) developed a scaffolding strategy so that students could work on complex situations—although concrete mechanisms to create such downscaling/upscaling activity sequences have not been defined yet.

Theoretical Framework

The learning of mathematics in the early stages must be tackled using the whole range of content, from the concrete to the most abstract, using realistic contexts as a vehicle for the development of abstract concepts by working in concrete settings that promote free constructions and productions by students during teaching/learning processes (De Lange 1996). During the process of learning mathematics, students should have the opportunity to develop and apply mathematical concepts and procedures in situations that are relatively close to their own reality, so that they can make sense of abstract concepts (Gravemeijer 1994). These educational methodologies, based on the study of realistic phenomena, have proved an effective way of helping children develop mathematical competence (Papadakis et al. 2017). And this can be further developed if combined with the use of educational technologies to deal with real information (Zaranis et al. 2013).

In this study, the M&M perspective is chosen because it has two essential characteristics that distinguish it from other theoretical perspectives (Abassian et al. 2020). These characteristics are: (i) the definition of the activities used to promote mathematical modeling, and (ii) the specification of the definition of mathematical modeling as a system of concepts and procedures.

In this way, the learning of mathematics can be promoted by solving MEAs (Doerr and English 2003) that connect with the students' reality or the close environment. Fermi problems (Ärlebäck 2009) have proved to be useful in getting students of different ages to deal with complex problems (Ferrando and Albarracín 2019; Ärlebäck 2009; Czocher 2016; Peter-Koop 2009). Estimation serves to overcome the difficulties generated by the complexity of the task. At the same time, by organizing the main problem into sub-problems and presenting them as a sequence of modeling activities (Ärlebäck and Doerr 2015; Pla-Castells and Ferrando 2019) it is possible to adapt the activity to the different students' realities and mathematical knowledge. The result of this learning is a better understanding of the phenomena under study and the mathematical models generated during the activity.

In this research it is understood that a mathematical model is a conceptual system designed to explain or describe a specific phenomenon. The mathematical model includes the concepts that shape the model and the appropriate procedures for solving the problem (Lesh and Harel 2003). From the point of view of obtaining direct information from students, it is difficult for them to adequately express the concepts that make up the models they create. From this perspective, student-generated mathematical models can be interpreted as what Steffe and Thompson (2000) understand as the students' knowledge of mathematics, which is the mathematical reality that researchers presume the students are familiar with, something which is inaccessible to outsiders. Therefore, researchers cannot rely on data that directly informs the concepts and it is necessary to use alternative methods. In previous studies focusing on models generated by students of different ages, we developed an analytical tool based on the definition of the mathematical model established by Lesh and Harel (2003) to characterize students' mathematical models according to their written productions when solving a Fermi problem (Gallart et al. 2017). Using this tool, the analysis consists of identifying the chains of procedures described by the students in their solutions and deducing the concepts that led them to choose those specific procedures. It is possible to retrieve the mathematical model used because students' actions are interpreted as a way of understanding the phenomenon and they choose the procedures they consider useful to fit the concepts that describe it. Thus, the action plan executed as a succession of procedures makes the mathematical model clear.

Methods

This is a teaching experiment (Steffe and Thompson 2000) that explored the role played by estimation in the mathematical models developed by second-grade students (7 to 8 years old) who were trying to explain the differences in population between a city and a town.

The researcher developed a sequence of estimation-based Fermi problems (Ärlebäck 2009) designed to scaffold students when comparing the population of a city and a town. During the implementation of the activity, the researcher adopted the role of observer and took notes on key interventions by students and the teacher. Finally, these interventions and the students' productions were analyzed from the standpoint of mathematical modeling. The following subsections provide more details on each part of the methodology.

Design of the Activity

The project designed to compare cities and towns had not contained any mathematical activities in previous years. The researcher proposed the reorientation of a project that had been implemented exclusively by the teacher up to that point. To this end, new mathematical activities were designed to work with on the project, based on proposing a sequence of Fermi problems to students using a downscaling and upscaling Fermi problems design strategy (Pla-Castells and Ferrando 2019). In this way, since it was assumed that the key problems posed by the project (How many people live in a city? And in a town?) were too complex for the students, some problems that they were able to solve by their own methods were presented first. First, the initial count was downscaled so that each student had to generate his/her own mathematical model by estimating the number of people living on his/her street. Then the count was upscaled to estimate the number of people living on all the students' streets. In this way the students were finally able to estimate the total population of the city. A similar process was followed for the population of the town, using Google Maps to obtain the required information.

Participants

The data was obtained from the work of a natural group of 24 s-grade students (7 to 8 years old) in conjunction with the classroom activities developed by the teacher. The parents approved their children's participation in the study and gave their written consent.

Activity and Data Collection

The aim of the project was to estimate the population of the city where the students lived as well as the population of a town they would later visit on an excursion. The main goal was to quantitatively compare them and give meaning to the quantities that emerged during the project. The teacher decided that several introductory questions were needed to guide the students' work by way of a scaffolding to solve the initial problem. Thus, it was the teacher who began the work of breaking the initial problem down into sub-problems, leaving an appropriate workload for the students. And it was also the teacher who took the first steps with the Fermi estimation method, leaving students with simpler problems that still needed to be broken down.

The classroom dynamic was as follows: the teacher presented the question to be answered and the students tried to answer it by working individually in the first activity and in groups of four in the following activities. There were several discussions involving the whole group, some to decide on ongoing methods and others to share and validate results.

To estimate the population of the city, the activity started with the question “How many people live on my street?” In a classroom discussion, the students decided to collect information about the type of buildings in the street where they lived and the number and quantity of dwellings in each building, and they agreed on a common procedure for this data collection. In the following session they discussed the number of people living in a dwelling and how to use various strategies to estimate the total number of people living on each of the streets. After the discussion with the whole group, the students worked individually using their own data. The next step in the project was to determine the total number of neighbors of all the students in the class. They discussed whether the value they had obtained was close to the total population and the teacher proposed using Google Maps as a tool to represent the streets where the students in the class lived, and visually compare this representation to the whole city. This activity provided an estimate of the total number of people living in the city. The next activity focused on estimating the number of people living in the town under study. An analogous procedure was carried out, but without visiting the town. This time they used Google Street View to estimate the number of people living on each street in the town as a step towards estimating the total population of the town.

During the students' work, an audio recording of the discussions involving the whole class group was made. The researcher also took notes in the classroom on the basis of an observation grid. The students’ written productions were collected after each activity. For the purpose of properly interpreting these productions, each working group was interviewed by the researcher to obtain a spoken explanation.

Analysis

First, the students’ written productions were qualitatively analyzed to identify the mathematical models generated by the students using the analysis tool presented in Gallart (2017). This analysis tool served to reconstruct the model generated by the students through the identification of the procedures used to solve the problem. Cobb and Whitenack (1996) define a mathematical learning opportunity as a situation where students have the opportunity to reorganize their conceptual structures or their ways of acting when they solve problems or, in general, where they have to cope with a novel mathematical activity.

Later, the activities the students worked on while generating these mathematical models were reviewed, including the statements, the researcher's classroom notes, and the audios of the activity, in order to identify the role of estimation in promoting mathematical learning opportunities and as a facilitator of the task of generating models.

Results

This section details the results derived from the project and how they respond to the research goals. To this end, the mathematical models generated by the students during each activity were presented, as well as the learning opportunities related to the modeling, and these were matched up with the specific role of estimation as a promoter of the activity.

People Living in my Street

To determine the number of people living on a street, the students' first proposals were based on exhaustive counts. In the full-group discussion, the teacher pointed out the limitations involved in carrying out these counts and explained to the students that it was not necessary to obtain an exact value, that an approximate value was good enough. This intervention from the teacher prompted the students to generate workable processes without the need for completely accurate responses. These processes supported the learners' models for representing the context of the problem. During the activity, the teacher collected the students' ideas and provided them with materials specifically developed to help them implement the procedures they had designed.
As detailed below, all the students developed models based on determining two relevant quantities: on the one hand, the number of homes on a street, and on the other hand, the number of people in each home. The differences between the students' productions depended on the specific procedures they used to achieve these values and how they combined them to obtain the result.

Tables of Multiples

Five of the students proposed a way to directly count the people living on their street without explicitly using the number of dwellings. These students took a reference value for the number of people living in a dwelling. When they presented the method to the class, the teacher agreed that they should be provided with incomplete multiple tables to be filled in first before using them for their street count later. Figure 1 shows part of the table of multiples of two used by one of the students. Thus, once the table was completed the students only had to count as many boxes as there were buildings on their street to get the answer.

2	4	6	8	10
12	14	16	18	20
22	24	26	28	30
32	34	36	38	40
42	44	46	48	50
52	54	56	58	60
62	64	66	68	70
72	74	76	78	80
82	84	86	88	90
92	94	96	98	100

Fig. 1. Part of a table of multiples of two used to count people who live in a street

Repeated Addition of Residential Buildings

Thirteen students followed a method in which they added up the number of residential buildings on their street as many times as the number of people they thought lived in them. Figure 2 shows the work of a student who counted 251 buildings in his street and took the number of 10 persons per building as a reference, arguing that in his street some of these buildings were individual homes and others were buildings consisting of several dwellings. In this case, the student's initial proposal was to add 10 a total of 251 times, which was the way he understood the distribution of people in the street. However, he asked the teacher if there was any concrete way to speed up the addition process. This represented a clear mathematical learning opportunity because it introduced students to multiplication through the need to use a method they had not yet studied.

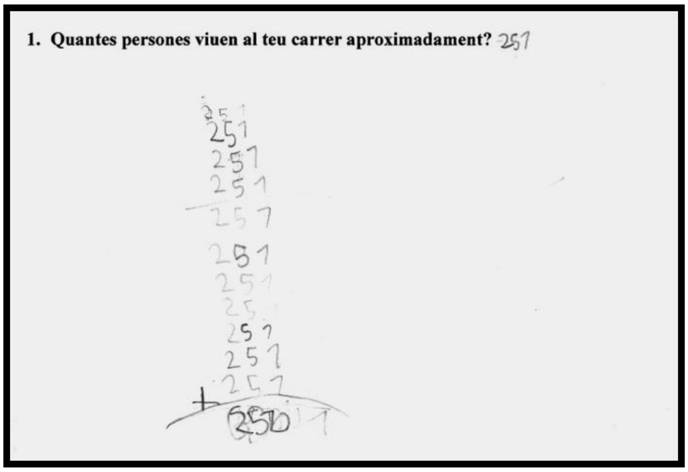
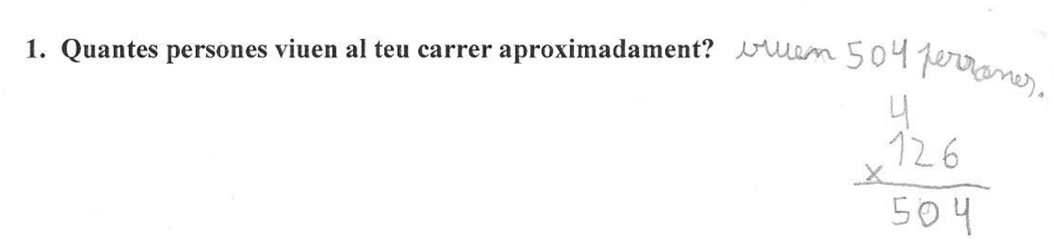


Fig. 2. Counting people living on a street from iterated sums

Multiplication

Six other students followed a similar path by choosing multiplication, whose factors were the estimated number of people living in a building and the total number of buildings on the street (Fig. 3). These students used a calculator to obtain their results. The fact that multiplication had not been studied previously in the classroom shows that school mathematical knowledge is complemented by knowledge acquired in other areas. In this case, students said they knew that they would learn the multiplication tables during the same school year because parents, siblings, and other students had told them.



1. Quantes persones viuen al teu carrer aproximadament? viuen 504 persones.

$$\begin{array}{r} 4 \\ \times\ 126 \\ \hline 504 \end{array}$$

Fig. 3. An example of the introduction of multiplication into the resolution process

People Living in my City

In this activity the students developed a single common model to estimate the number of people living in their city. The reason for this was that initially they worked in groups of four to draw up their proposals, but they observed the need to share the partial results generated in the previous activity. Therefore, they considered it necessary to work all together. In this case, the students marked the number of people living in the streets where they live on a map of their city and colored these streets yellow. In this way they first determined the number of people living in their neighborhood by adding up the figures obtained for each student's street. In a second step, and using a map of the whole city, they visually determined that the city was 10 to 12 times larger than their neighborhood. This was an activity where the role played by estimation was to establish a relationship of proportionality between two sets, which was finally solved by multiplication. Again, students used a calculator to obtain the result.

People Living in a Town

The activity “How many people live in a town?” took the students back to the models generated in the first two activities but introducing some different characteristics. Again in groups of four, the students concluded that it was necessary to know how many streets there were in the town to solve this question, and therefore, using Google Maps and various photocopies of maps, they proceeded to count. Some trusted their memories, others marked the streets they had already counted with some kind of sign, and a few chose to number each street, thereby achieving a more systematic and reliable result. Figure 4 shows this numbering of streets on a map of the town.

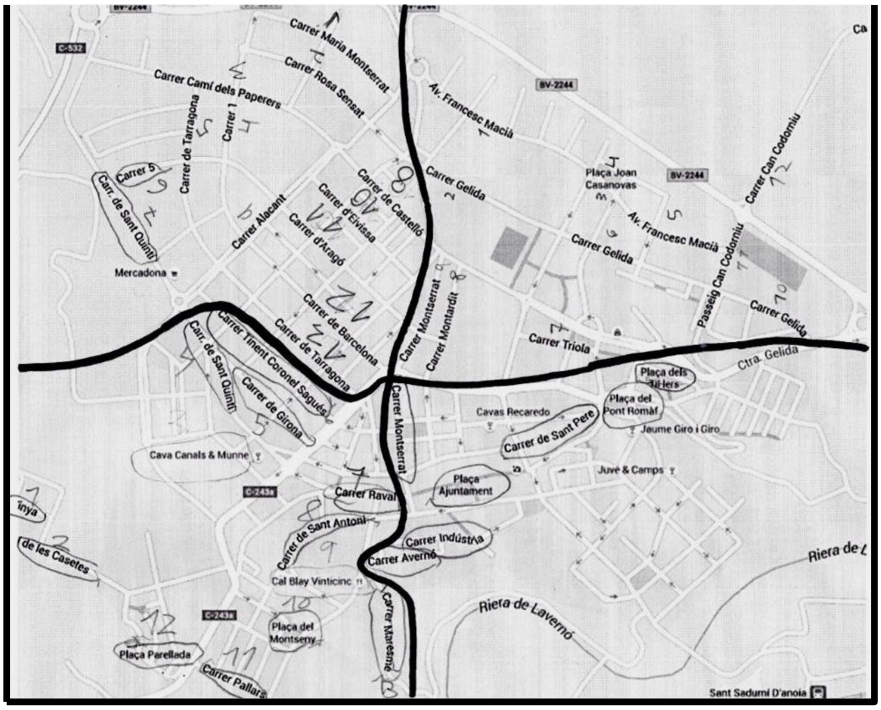


Fig. 4. Sample showing how streets were counted in the town by one of the groups

Google Street View was also used to estimate the approximate number of residential buildings per street, as students observed in the street-level views that the same parameters could not be applied in a city. Out of the six groups of students in the class, four chose to use multiplication, although the linking together of various operations resulted in several calculation errors, and two chose repeated sums. It should be noted that working with large numbers led some of the students to realize the need for multiplication as a way of optimizing the strategy they were carrying out. Figure 5 shows the work of a student who tried to add the number 106 (houses in one street) 48 times (the streets) and ended up multiplying on the advice of a classmate. Finally, he multiplied the result obtained by 2 (people in each home).

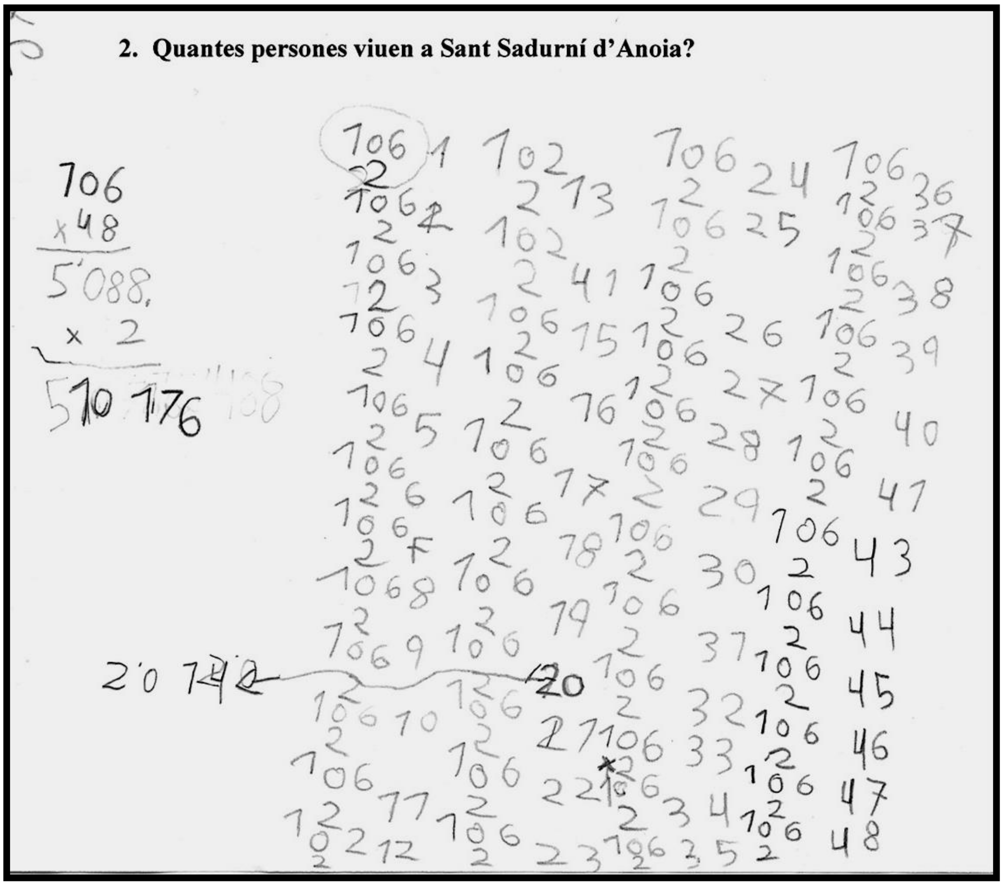


Fig. 5. Necessity of using multiplication for the iterated sum when counting people in a town

Comparing a City and a Town

Once the students had completed their estimates for the number of people living around them in the city, and the population of the town had been studied, it was time to compare these two values. The result obtained for the total number of residents in the streets where the students live was 15,750 people. At this point a student noted that none of the values obtained was accurate and that this possibly jeopardized the validity of the final result. Seven other students agreed with this opinion, which was not appreciated by the rest of the class. In fact, they did not seem to understand the implications of the observation. The teacher argued that none of the values obtained could be exact, and, given that there would be both overestimates and underestimates, the errors had to be balanced out. She proposed a range that the students could find credible, and they agreed that a value of between 15,000 and 17,000 was very plausible.

For the total population of the town, the students' estimates were all between 8500 and 11,000 people. In this case the students discussed the accuracy of this figure. At first, some found it hard to accept the values proposed by others, but the teacher pointed out that it was not necessary to obtain an exact result and that a reliable range of values was adequate. The students agreed that the result should lie between 9000 and 11,000.

A direct comparison of these two sets of values showed that a city was larger than a town, but these figures on their own did not indicate the real value of this difference. Therefore, the teacher's proposal was to highlight all the streets where the students lived on a map of the city. In this way, they observed that together they covered a small proportion of the whole area of their city and that they were able to extrapolate their results to the population of the town they were comparing it with.

Discussion and Conclusions

This research focused on identifying mathematical learning opportunities in the form of mathematical models (Lesh and Harel 2003) promoted by the use of the estimation of large numbers by second-grade primary school students. To do this, an activity based on solving a Fermi problem (Ärlebäck 2009) was used to compare the population of a city and a town. Previous studies show that the use of Fermi problems promotes student competency in the development of mathematical models, whether in primary education (Albarracín and Gorgorió 2018; Peter-Koop 2009), secondary education (Ärlebäck 2009; Albarracín and Gorgorió 2014) or at university level (Czocher 2016). The results presented in this article extend the age range in which estimation using Fermi problems facilitates mathematical modeling processes, with second-grade primary school students participating in the creation of mathematical models to address complex real situations. To this end, it is necessary to consider the specific role of estimation as a facilitator of students’ mathematical work and also two basic aspects involved in the design of activities. These key components are discussed below.

First and foremost, this study shows that 7- to 8-year-old students can develop their own methods for estimating the number of people living in a city and a town. These methods are based on different methods of counting that attempt to answer questions where an estimate is an adequate response. The products developed by the students constitute mathematical models as defined by Lesh and Harel (2003). Therefore, Fermi problem-based activities are model-eliciting activities (MEAs) for second-grade students when they focus on working with situations close to the students, and model exploration activities (MXAs) when working with faraway contexts.

During the project it was observed that estimation facilitates mathematical activity in different ways. First, when students tried to answer accurately, they only proposed exhaustive counts, something that depended on information that was difficult to obtain and was very time consuming, and which they could not carry out by themselves. However, as soon as the need for accuracy was eased, their approach to the problem changed and they began to consider various alternatives. Thus, mathematical learning opportunities emerged (Cobb and Whitenack 1996), such as the use of multiplication to calculate repeated sums or the proportion factor in a subset and the total set. Students ended up developing simplifications of reality that facilitated the process of mathematization and, finally, the production of a mathematical model.

In reference to the project design and the activities proposed to the students, two key factors were identified that helped students to address the main problem. The first was that removing the need to obtain concrete values and then working from estimates unblocked the students, because this made the activity feasible. During the activity, the teacher had to explain to the students that an exact result was not required because the goal was to obtain a reasoned approximate value that would serve as an estimate of the result. The second factor was that the teacher reduced the complexity of the task by proposing the main project and accompanying it with a suitable division into various problems that were more accessible to the students. In this way, the teacher was able to carry out a prior analysis of the problem to be solved and determine a first level of sub-problems so that each task was accessible to the students (Carlson 1997). This resulted in a modeling activity sequence (Ärlebäck and Doerr 2015), while ensuring that there was still room for them to analyze the proposed situations and to break them up into simpler problems.

Thanks to these two conditions the second-grade students were able to tackle the study of the proposed real phenomenon by generating simple but effective mathematical models to estimate the number of people living in their neighborhood and living in the selected town, by establishing methods for their comparison. Furthermore, this activity design enabled students to break down each of the problems presented to them into sub-problems, working in the same way as proposed by the Fermi estimation method. At this point, estimation played a new key role, because the students assigned reasoned values—those they considered appropriate—to quantities such as the number of people living in a residential building. This task design is shown in Fig. 6, where the teacher proposes the first level of sub-problems and the students use the Fermi estimates method in the second level of sub-problems.

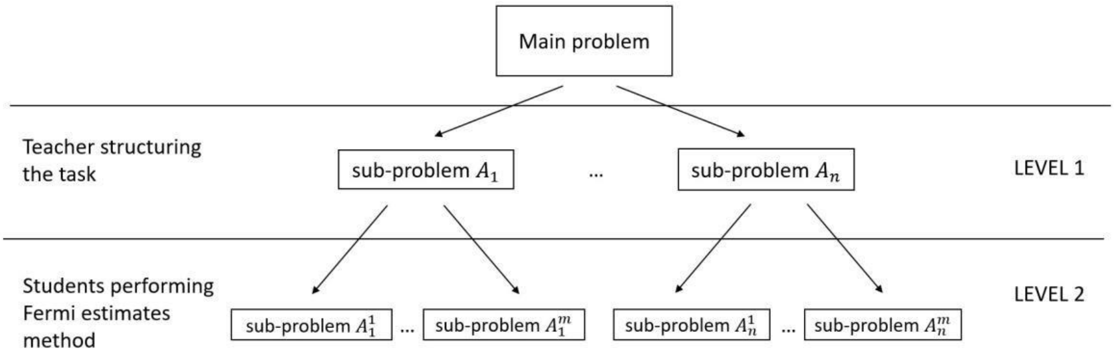


Fig. 6. Design structure of the project

This organization of the activity is the concrete expression for this type of projects of the scaffolding proposal developed by Pla-Castells and Ferrando (2019), focusing on a larger concept when solving each problem (Lesh et al. 2013).

To sum up, this study establishes that projects based on estimating large quantities using the Fermi problems help to get second-grade students started on the development of mathematical models. It paves the way for the study of other relevant social contexts that can be addressed by students in the first years of primary education and for further exploration of how the use of estimates engenders learning opportunities related to mathematical modeling.

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