

Mathematical learning in a context of play

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ABSTRACT: In this article we analyse a didactical situation centred on the creation and use of a symbolic play environment in a class of pupils aged five and six years-old. The main source of data for this paper comes from an experimentation planned in relation to the following research question: does symbolic play in simulated contexts help pupils to increase their mathematical activity in preschool learning? We draw on the sociocultural teaching and learning theories inspired by Vygotsky. We argue that the creation of and participation in a simulated context in classroom play is an adequate starting point for working on situations of social interaction and for promoting a joint construction of mathematical knowledge. The findings show the evolution of the pupils' actions during the symbolic game and the progressive incorporation of concrete arithmetical contents into their mathematical thinking.

RÉSUMÉ: Dans cet article, nous analysons une situation didactique centrée sur la création et l'utilisation d'un environnement de jeu symbolique dans une classe d'enfants âgés de cinq à six ans. La principale source de données provient d'une expérimentation conçue pour répondre à la question de recherche suivante: est-ce que le jeu symbolique, dans des contextes de simulation, aide les élèves à accroître leur activité mathématique au sein des apprentissages préscolaires? Nous nous appuyons sur les théories socioculturelles de l'apprentissage et de l'enseignement, inspirées par Vygotsky. Nous avançons que la création et la participation à un contexte de simulation en classe constituent un point de départ adéquat pour travailler sur des situations d'interaction sociale et pour promouvoir une construction conjointe de connaissances mathématiques. Les résultats montrent l'évolution des actions des élèves au cours du jeu symbolique et l'incorporation progressive de contenus arithmétiques concrets dans leur pensée mathématique.

ZUSAMMENFASSUNG: In diesem Beitrag wird eine didaktische Situation analysiert, die sich auf die Schaffung und Nutzung einer Umgebung für symbolisches Spiel in einem Klassenzimmer für fünf und sechs Jahre alte Schulkinder zentriert. Die Hauptdatenquelle dabei stammt aus einer Versuchsdurchführung, die in Hinsicht auf folgende Forschungsfrage geplant wurde: Hilft symbolisches Spiel in simulierten Kontexten Schulkindern dabei, ihre mathematischen Aktivitäten im Vorschul-Lernen zu steigern? Wie beziehen uns auf die von Vygotski angeregte Theorie soziokulturellen Lehrens und Lernens. Wir argumentieren, dass die Schaffung von und Partizipation in einem simulierten Kontext im Spiel innerhalb des Klassenzimmers ein angemessener Ausgangspunkt für das Arbeiten an sozialen Interaktionssituationen und für die Förderung einer gemeinsamen Konstruktion mathematischen Wissens ist. Die Ergebnisse zeigen die Entwicklung der Handlungen der Schulkinder während des symbolischen Spiels und die fortschreitende Einarbeitung konkreter arithmetischer Inhalte in ihr mathematisches Denken.

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RESUMEN: En este artículo analizamos una situación didáctica centrada en la creación y uso de un medio de juego simbólico en una clase de alumnos de 5 y 6 años de edad. La fuente principal de información para este artículo proviene de un experimento planificado en relación a la siguiente pregunta: ¿Ayuda el juego simbólico en contextos simulados a mejorar la actividad matemática en el aprendizaje preescolar? Partimos de teorías de enseñanza y aprendizaje inspiradas en Vygotsky. Argumentamos que la creación y participación en un contexto simulado en el juego de la sala de clases es un punto de partida adecuado para trabajar con situaciones de interacción social y para promover la construcción conjunta del conocimiento matemático. Los resultados muestran la evolución del accionar de los alumnos durante el juego simbólico y la incorporación progresiva de contenidos aritméticos concretos en su pensamiento matemático.

Keywords: mathematics education; sociocultural theories; early years; play; social interaction; simulated context

Introduction

We draw on sociocultural research (Planas and Edo 2008) in the area of mathematics education (Lerman 2000) at an early age (Clements, Sarama, and Di Biase 2003). We do this by integrating the notions of play, social interaction and simulated context. The interest in the topic of mathematical thinking in early years has increased immensely, as the recent appearance of different international groups such as the 'Early Years Mathematics Group', of the European Society for Research in Mathematics Education, proves. We share with this group the assumption that the learning of mathematics depends on the pupils' participation in processes of collective construction of reality (Krummheuer 2007).

Although our work is interdisciplinary in many respects, we mainly focus on mathematics education and foster research questions with regard to mathematics learning. In particular, the main source of data for this paper comes from an experimentation planned in relation to the following research question: does symbolic play in simulated contexts help pupils to increase their mathematical activity in preschool learning? We do not present symbolic play in relation to other didactical situations and, therefore, we cannot compare the increase in the children's mathematical activity as being developed to a far greater extent than in other situations. Instead, the focus of the research is on the exploration of some of the characteristics of this activity during a particular type of non-traditional teaching.

Our aim is to create and analyse a contextualised didactical situation, associated with out-of-school realities, that involves students aged five and six years in a cooperative and shared experience concerning the learning of mathematics. We organise the didactical situation by relating the learning contents to the context of a shop. From the perspective of the simulated context and when considering methodological issues, Pramling's classical study (1991) about the development of children's understanding of a shop is similar to our current work. However, this author did not pay special attention to mathematical knowledge but to particular aspects of the children's lived experiences; neither did she identify the acquisition of concrete mathematical procedures and notions in the evolution of the pupils' actions. Like Pramling, we assume that the child's understanding of the world around her/him requires a global knowledge, but in our work we argue that this knowledge can be partially explored by searching for disciplinary contents such as those related to mathematics.

We begin with a short theoretical discussion on some key notions and then move on to the orchestration into practice of them through the design, development and analysis of a classroom didactical situation that serves as the scenario for collecting data.

Theoretical framework

The relationships between play, learning and development have been extensively studied (Piaget 1962; Vygotsky 1978). Elsewhere (Edo and Revelles 2004; Edo and Deulofeu 2006; Edo 2008) we have argued the importance of contexts of play in the study of the teaching and learning processes of mathematics in early years and primary schools. Our own work and the existing literature on these topics, however, is not still sufficient, as Van Oers (1996) states:

I draw the cautious conclusion, that play activity can be a teaching/learning situation for the enhancement of mathematical thinking in children, provided that the teacher is able to seize on the teaching opportunities in an adequate way. To what extent this approach also leads to lasting learning results in all pupils is an issue for further study. (71)

Van Oers (1996, 1999, 2003) considers symbolic play to be an activity that enhances learning at an early age. In this author's work, it is assumed that the children's learning processes should be embedded in their play activity, and that these learning processes could be positively based on the children's ways of dealing with symbols, symbolic representations and meanings. In particular, the activity of symbolic play can be seen as providing the general framework in which different mathematical actions are developed. Symbolic play requires a high level of freedom in the learner's action processes; in the school context, this autonomy must be combined effectively with the teachers' intervention in order for the play to evolve properly (Van Oers 1994). In order to guarantee this evolution, the learners are supposed to incorporate progressively the elements of reality introduced by the teacher into their symbolic play, it being the teacher who creates stimuli for said evolution without having a negative effect on the pupils' free play.

In Edo and Deulofeu (2006), we argued that classroom play environments tend to favour social interaction and communication. In that work we interpreted learning to be the result of participation in interactive contexts (Rogoff 2003). We associated the learning of school mathematics with the development of interactions that make it possible to communicate and appreciate the different points of view of the participants in the class, and thus complete the meanings that have been learned. From this perspective, the teacher and the pupils interactively construct the classroom culture by means of the processes of constructing shared meanings. Perret-Clermont, Pontecorvo, Resnick and Zittoun (2004) indicate that social interaction makes it possible for people's subjective knowledge to become 'objective' knowledge, which is what forms the basis for the content of classroom culture. Burton (2003 in Göncü and Perone 2005, 44) also refers to the significance of social interaction as a way of conforming 'objective' mathematical knowledge. The interactions in a class serve as conditions needed for the construction of shared and public 'mathematical stories' that, in turn, serve as a discursive-based approach to mathematical learning and teaching environments.

Edo (2005) has shown how the participation of pupils in simulated contexts and real contexts is an efficient form in which children can develop mathematical

problem-solving strategies. In the literature on mathematics education and the area of the so-called realist current, De Lange (1996) has introduced the notion of simulated context. The origins of the simulated context lie in the real context and reproduce some of its characteristics (for example, when the pupils simulate buying and selling contexts in a corner of the classroom). In the infant classroom, the simulated context is especially important given that it makes it possible to imagine real situations and consider them from the perspective of mathematical models (Gravemeijer 2007). Here, knowing about mathematics is related to skills that involve applying mathematics to real-life situations and to other situations than those that are represented in the classroom.

When considering Van Oers' work (see, for instance, Van Oers 1996), we share the focus on the mathematics and the symbolic play but we differ in the use of simulated contexts. Despite this difference, some of this author's findings are central to our current research. In his research with young children, Van Oers has proved that teaching and learning opportunities occur within role-play activities. He concludes that play activity might be a teaching and learning situation for the enhancement of mathematical thinking in children. One of the didactical challenges is then to help the development of the pupils' thinking without interrupting them during the play activity. If considering a research point of view, one of the challenges is to gain more data on this type of didactical situation, and to do it in relation to the acquisition of specific mathematical contents. Hence our data may be interpreted as an effort towards the complementation of Van Oers' and Pramling's findings, as well as our own previous findings.

Context and method

Our research was carried out with a group of 26 pupils aged five and six at a public urban school near to Barcelona, Spain. The didactical situation was based on a symbolic play area in the corner of the classroom. The aim was to construct a simulated context in the classroom in order to help the pupils learn some arithmetical contents and to use them integrated into other contents. It was thought of as an interdisciplinary didactical situation, focused on the improvement of out-of-school knowledge, the pupils' capacity for working co-operatively, and the use of the mathematical language being learned. Other important didactical goals were to:

- (1) promote observation, analysis and learning in a global real-life context;
- (2) organise the collaboration and cooperation of the pupils, as well as the processes of making decisions, sharing tasks, coming to agreements, participating in groups for a common end, etc.; and
- (3) use cultural tools (reading and writing, oral skills, drawing, modelling, etc.) in real-life and school contexts.

Table 1 summarises the 13 sessions that conformed the didactical situation. First there was a group of preliminary activities (sessions 1 to 7) that were planned in order to create the play corner. Second, we had the first game on buying and selling (session 8), which was expected to happen without the teacher's intervention except for the initial assignment of roles. Between the first and the second game, there was a first group of complementary activities (sessions 9 and 10) that introduced the use of two cultural tools – calculator and money – in a non-play situation. In the second game

Table 1. The activities of the didactical situation.

| Type of activities | Sessions |
|--------------------------|--|
| Preliminary activities | 1. What is a shop? 2. What shop do we want to make? 3. What do I imagine the bakery will be like? 4. What do we need? 5. Visiting a bakery 6. What will we call our bakery? 7. Making the shop |
| Game I | 8. Playing at buying and selling |
| Complementary activities | 9. Finding out about the calculator 10. How do we add up five euros? |
| Game II | 11. Playing at buying and selling |
| Complementary activity | 12. 'Making up'; sums with the calculator |
| Game III | 13. Playing at buying and selling |

(session 11), the teacher only intervened at the beginning when establishing that each buyer would have €5. Before the third game started, there was another complementary activity (session 12) that appeared as a demand coming from the children: it consisted of inventing additions with the calculator. The last game (session 13) was again about playing at buying and selling with €5 each children, but now the teacher wanted them to work together when making a list and buying things.

Van Oers (1996) claimed that symbolic play requires a high level of freedom in the pupils' actions that must be respected by the teacher's actions. According to this author's assumption, our didactical situation was planned and developed so that during the time for symbolic play the teacher did not intervene. The preliminary and complementary activities – and the initial instructions given by the teacher only at the beginning of each game – are expected to ensure that the pupils' play remains free. From the point of view of the mathematical thinking, the play is expected to gain in complexity without having needed the teacher's interventions during the interactions among pupils.

We had collaborated in prior research with the teacher in the classroom and the school. This school was reform-oriented in the interpretation of the curriculum and the teachers were used to working collaboratively. The children in the classroom came from a variety of socioeconomic backgrounds and they had been organised as a regular group two years ago. During the time for mathematics, they had not been previously involved in symbolic play activity; instead, they had worked on exploratory play with materials and paper-and-pencil activities. From the perspective of the teacher's participation, the study was carried out using a research-action methodology. This methodology helped the teacher to conceive her work as that of a reflexive professional able to generate transformations, both in her professional knowledge and in the social context she was intervening in. We considered the teacher to be one more member of the research team, in that she contributed to the decision-making and was a key player in the process of triangulating perspectives. The research team was completed by a group of teachers at the school and the authors of this paper. The 13 sessions distributed in four weeks that made up the

didactical situation were conducted by the same teacher, sometimes accompanied by a support teacher.

The data was collected by means of classroom observations, classroom audio and video-tapes, interviews and written productions. The first author observed the 13 sessions and took field notes. We had also made audio and video recordings of the sessions devoted to symbolic play. These recordings were transcribed and analysed as evidence of the development of interactions in terms of the construction of shared meanings. Moreover, each pupil was interviewed on two occasions: at the beginning and at the end of the didactical situation. The evolution in the pupils' oral responses and explanations was completed with the evolution in the written productions, which were taken on three occasions, one for each moment devoted to symbolic play (sessions 8, 11 and 13). The analysis was carried out on the basis of the classroom transcripts and the pupils' written productions. For each child we elaborated an extended written document with part of the field notes, some of the narratives from the interviews, some of the episodes from the classroom transcripts and all of that pupil's productions. After having elaborated 26 individual documents, we developed a systematic search within each of them. We looked for mathematical contents related to five categories that expressed arithmetical skills that had been used, referred to or somehow represented by the pupil during the time devoted to the didactical situation (sessions 1 to 13). The categories and our interpretation of them were the following:

- Quantitative use of non-numerical words. The children use non numerical quantifiers such as all, some, much, many, few, none, less than, more than, as many as etc. that point to issues of quantity in their oral and/or written discourse.
- Use of numbers without quantitative correspondence. The children use number-words in their oral discourse or represent written numbers that appear as related to prices, changes, and quantities of objects that have been bought. They do it in a simulated way, within the game, without looking for a real correspondence with the quantitative significant of the number.
- Use of numbers with quantitative correspondence and non-arithmetical meaning. The children use number-words in their oral discourse or represent written numbers that are related to prices and quantities of objects that have been bought, without making operations.
- Arithmetical use of numbers. The children use number-words in their oral discourse or represent written numbers that are referred to the correct result of an operation that has been carried out during the actions in the play situation.
- Use of numbers with calculator. The children use the calculator in a functional way during the game in order to solve or check the result of an operation that has a direct relationship with an action that they have developed.

For the analysis of data concerning these five categories in the case of each student, we paid special attention to the early representational activities of the children such as drawing, writing to drawings, and talking about representations. The main empirical data that we present in this paper are referred to representational activities.

The teacher's instructions during the game sessions are open enough for the children in order to let them choose what they prefer to comment on in relation to the general actions of buying and selling. They are not asked, for instance, about their expenses or their buying; neither are they asked to represent their actions with concrete representations such as words, numbers or drawings.

Results

We organise some of the results of the study in the chronological order of the didactical sequence that was implemented by the teacher (see Table 1). We explain the different moments of this sequence by highlighting the following:

- (1) the process of joint construction of a simulated classroom context (mainly sessions 1 to 7); and
- (2) the evolution of the participation and the mathematical learning in symbolic game situations (mainly sessions 8 to 13).

We present the results, with some of the pupils' narratives and representations, as a narrative in order to facilitate the examination of the many different ways through which the children in the class were socialised into both the class and the teaching and learning situation.

Preliminary activities

Session 1: What is a shop?

The teacher opened with a preliminary conversation with the group, getting them to answer questions such as what is a shop, what is a shop for, and what do we need to buy in a shop. All of the references the pupils came up with were related to one specific type of shop, such as, for example: 'A place where you go to buy things... things to eat...', 'There are different shops for buying clothes', '...if they want to buy some medicine', '...to buy bread', etc. The teacher maintained a dialogue on the last question, 'What do we need to buy?', with some of the pupils. They only mentioned food, clothes, medicine... This group conversation, together with the dialogues with the pupils, helped to make an initial evaluation of the group's background knowledge. The children's narratives reflected their more spontaneous understandings of a shop as a particular type of shop with objects in it that could be seen and bought. It was first seen, however, that the concepts of price, money and exchanging money did not come up.

Session 2. What shop do we want to make?

In order to decide the kind of shop, the children were asked to come up to the board and write their answers (with the help of all their peers, as some were at an early stage of learning to read and write). Some of the words they wrote did not correspond to any type of shop. With their comments, the children indicated the things that were not 'types of shops', but objects that you would buy in them. Four types of shop were left: ironmonger, shoe shop, bakery and pharmacy. They did not all support the same option and so a solution had to be found. One boy spoke authoritatively: 'Let's just make a pharmacy and that will do.' He quickly realised that he had peers who did not agree with him and it was eventually a girl that said: 'What about voting on it?' This option was accepted given that one of the aims of the task was for the pupils to find a way of finding shared solutions to the problems they came across. This 'good solution' to a conflict offered the chance to use their mathematical knowledge in a functional context. Once they had divided into groups that supported each of the options, they had to count (as far as they needed) to find out which type of shop had won the

most votes. Then they wrote all the options down and drew a circle around the one that the class had voted to build: the bakery.

Session 3. What do I imagine the bakery will be like?

For the next activity, each pupil drew the bakery as they imagined it would look, in accordance with their prior knowledge. The teacher did not explain new content without asking the children first (collectively and individually). At the end, the different items of individual work were displayed, grouped and commented on. Figure 1 shows some of the results: to the left, a boy draws the bakery he usually goes to and names little bears, mini croissants, bags, potato chips, chocolate, doughnuts, *Kinder Eggs*, water, juice, savoury sticks, loaves of bread, chocolate balls etc. To the right, a girl's picture includes the classroom tables and chairs that will be used to make the bakery in class, and types of bread, buns etc.

Session 4. What do we need?

The next step was to find out 'what' was needed to set up our bakery. The activity was done in small groups of seven or eight children. Each group had to think and write about what they thought was required to make a bakery. The different groups organised the work differently. Some said aloud the names of the things to be written down, and passed the paper round as they worked out what letters were needed to write them. In other groups, all the children wrote at the same time and had no idea of what the other children next to them were writing. Others used pictures 'because they might not understand the letters'. At the end, each group commented on what the other groups had done and observed the variety of answers. After visiting a bakery near to the school (session 5) and coming back to class, they talked together about the things they had forgotten or that they had wrongly included in their lists. They realised that without money, a calculator (cash register), receipts, prices and paper they could never have a proper bakery in class. They completed their lists, this time writing down the things they had seen they were missing.

Session 5. Visiting a bakery.

When the children and the teacher visited a bakery, they watched a man buying something, and then they imitated what he had done (they bought four types of bread from

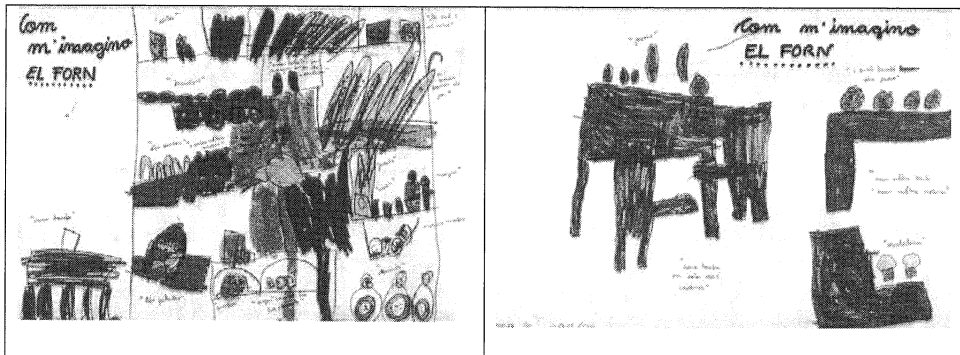


Figure 1. Pupils' representations of a bakery.

the ones the shop assistant showed them). They gave her money and she gave them back more money and a piece of paper. This situation was discussed back in class. They looked at the paper and tried to work out what it said. They saw there were numbers and letters. After several hypotheses, they decided they were the prices of the four types of bread and that the paper was called a receipt. They spoke about the numbers written next to the things being sold, and worked out that these were the prices of each product. Finally, they made a sheet summarising what had happened in the bakery and wrote down what they had learned. When making the written summary, the children made links between the simulated context and the context of reference that had been presented to them as real. These links were expressed by means of drawings with objects from both the classroom and the visited bakery.

Session 6. What will we call our bakery?

The children noticed that they did not have a name for their bakery. New technologies were used to come up with the name. Individually, each child used the computer to write the name s/he wanted to give to the bakery. These were then printed and placed on a sheet prepared beforehand. When each child had written their name, the teacher asked them what they thought they had to do next. The answer was to vote. The teacher wrote down all the options on the board and they all started voting. When they had settled on a name, which combined the two most voted options: 'Pa del fornet' ('bread of the little bakery'), each child wrote the name at the bottom of the pre-prepared sheet. They counted how many letters it had, cut out a piece of paper for each letter, so that each pupil could take part in writing the name of their shop. Meanwhile, some of the pupils made models of the objects that were going to be sold in the shop.

Session 7. Making the shop

To build the shop in class, some groups decided how the pieces of paper were to go on the walls and hung them up, others fetched tables and organised the positions of objects, and another group put the products to be sold in place. These were dry varnished bread and clay figures they had made. Before they started playing, they had to think of prices for the bread and other products. Their parents were involved in this activity. Each child had to take home a piece of paper with the names of four things they planned to sell in the shop and had to find out the real prices. In class, they spoke about the prices, what the numbers and decimal points meant, how to read them, whether they came to more or less than , etc. They noticed that some prices varied from shop to shop, but often the highest and lowest prices were the same. They then sorted the products out, from the most expensive to the cheapest. They decided the prices and made price lists. Each child wrote a price for one of the products using the agreed prices. The final prices were the following: bread, €5; a croissant, €3; an 'ensaimada' (a typical Mallorcan pastry), €2; and a bun, €1.

From Game I to Game III

Session 8. Playing at buying and selling. Game I.

The first time they played in the shop corner, the teacher gave them roles as buyers and sellers and no other orders were given. Groups of five or six children were freely

formed to go to the play area and to role-play. Most of them applied some of the contents from real life that they had learned after going to the bakery, but at this moment nobody used the calculator and references to euros and prices – whose use was experienced in the out-of-school visit – were hardly made. After each game, the children were asked individually to make a display of what had happened while they were playing with their small group in the simulated shop. The teacher literally said: ‘Explain your buying.’ They knew that they could ‘explain’ by drawing, writing words, writing numerals, etc. Here, the children’s representations were thought to provide information concerning both the children’s understanding and the teaching environment that would be more useful in the development of a more complex mathematical thinking. It was decided, for instance, that the children were given €5 for the development of the next play situation.

Figure 2 shows the display made by Ona at this stage. She drew and said aloud what she had bought: a loaf of bread, three buns, an ‘ensaimada’ and some cookies. The objects and quantities correspond to the activity but she did not mention prices or money in the oral and written explanation of her drawing. Table 2 illustrates findings concerning some of the pupils’ actions during the first game. We organise the data into five categories. When it is possible, we give an example of a pupil’s oral and/or written narrative. The narratives have been placed within the categories after having been examined its meaning within the more general discursive context of use within the classroom’s social interactions.

Session 9. Finding out about the calculator

The next activity was done in groups of four pupils, while the other students in the class were involved in different tasks. Each group was given one calculator and they were asked the following questions:

- How do you turn the calculator on and off?
- What do we see on the screen when we turn it on?
- Can we write how many children are in this class?
- How can we erase it?
- What do we see when we erase it?

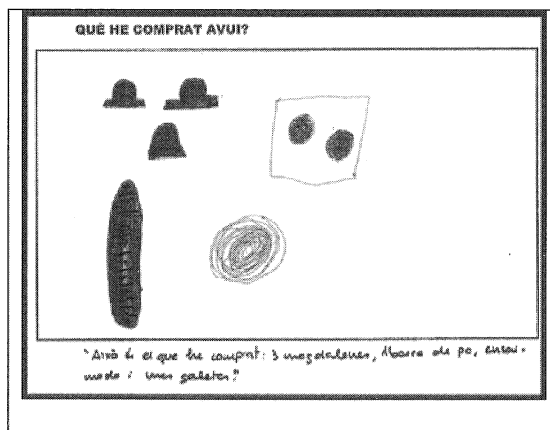
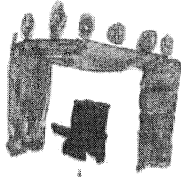


Figure 2. Ona’s representation in the first game.

Table 2. Findings related to Game I.

| Categories | Pupils' narratives | |
|--|---|--|
| | Oral narratives and actions | Written narratives |
| Quantitative use of non-numerical words | I have bought many buns, more than Claudia. | I have bought many breads, all are breads. |
| | |  |
| Use of numbers without quantitative correspondence | It costs two euros (she gives €2 and she's given back a €2 exchange). | — |
| Use of numbers with quantitative correspondence and non arithmetical meaning | I have bought a loaf of bread, three buns, an 'ensaimada' and some cookies (she talks about the numbers in her drawing) [Fig. 2]. | — |
| Arithmetical use of numbers | — | — |
| Use of numbers with calculator | — | — |

What does the number we see mean?

I went to buy a croissant and I paid three euros, and an 'ensaimada' cost me two euros: how much did I spend?

Can I work it out with a calculator?

After learning to handle a calculator, each child explained on a sheet of paper what they had found out or learned. Again, the teacher literally asked them to 'explain'. Most of them used symbolic representations of calculators, operations and prices, which made an important difference from the use of numbers that had been represented during the previous activity.

Session 10. How do we add up €5?

In this session, the children recognised the value of different real coins and notes. By working in groups and being guided by the teacher, they had to add up €5 in different ways. For this task, they were given a box with simulated coins. They all had different reactions with the variety of existing coins and in general they were more comfortable with the manipulation and use of the €1 and €2 coins. The teacher introduced the €5 note and asked them to look for groups of coins that added up €5. The work with the coins, as well as that with the calculators, was planned in order to introduce cultural artefacts that acted as mediators in the pupils' mathematical thinking and that somehow allowed the teacher not to intervene in the resolution of the given tasks.

Session 11. Playing at buying and selling. Game II.

For the second game, pupils were arranged into groups of four to six children, and times and turns were established for the activity. They started using the calculator,

always paid in euros, and often used paper-and-pencil strategies to make receipts and shopping lists. The pupils' symbolic game was still free but they were making increasingly greater use of the ways things are done in the real-life context. Here, the mathematics they had learned played a crucial role. Each of the children was given €5. They were told to be careful about what they were buying because they had to make sure they had enough to buy everything, and it didn't matter if they had too much money. They helped each other to count their euros and add up how much everything cost. More than half of the groups used paper and pencils to help them, to make shopping lists or to make receipts or add up the prices.

Figure 3 shows the display made by Ona. She drew exactly what she had bought: two buns, at €1 each, and an 'ensaimada' for €2. The total price corresponds to the money she could have paid (up to €5). She also drew four euros coins, showing the amount that she had spent. Figure 3 can be compared with the display in Figure 2, where this pupil does not make an arithmetical interpretation of the quantities that she represents. Table 3 illustrates some of the findings concerning the pupils' actions during the second game. The comparison of the data in this table and the contents summarised in Table 2 point to a clear evolution of the group towards the use of quantities with arithmetical meanings in simulated contexts that have been initially approached from a qualitative non arithmetical interpretation.

Session 12. 'Making up' sums with the calculator

The boys and girls liked the calculator session so much that they asked to do it again. The teacher asked them what they wanted to do with the calculator, some children suggested they could 'do sums', and the whole group agreed. A few weeks later some calculator sums were 'made up'. The class was arranged into groups of four with two calculators each. Like on previous sessions they worked in cooperative groups and, although the calculator was differently used in each group, most of the groups tended to give this tool an arithmetical function situated within the simulated context of the classroom shop. The value attributed to the play situation by the children was especially evident when most of them took the initiative to relate the task of making up sums with the calculator to the task of qualitatively interpreting the addends in the context of the shop.

Session 13. Playing at buying and selling. Game III.

For the third game, each child was given €5 again, but the teacher suggested that two or three children should join up to make a joint purchase, so now they could get

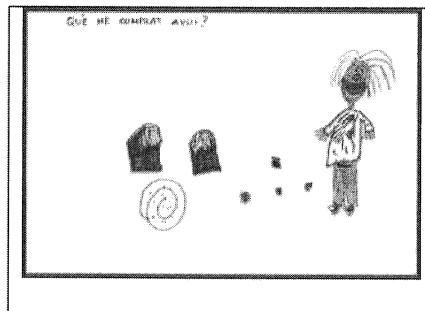

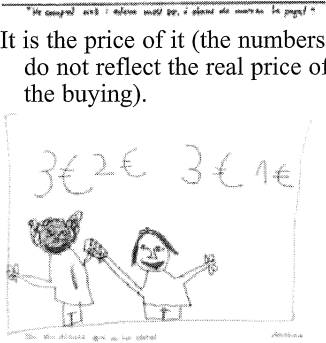


Figure 3. Ona's representation in the second game.

Table 3. Findings related to Game II.

| Categories | Pupils' narratives | |
|--|--|--|
| | Oral narratives and actions | Written narratives |
| Quantitative use of non-numerical words | I have bought as many buns as money I had. | I bought this and it was tasteful. Before leaving I gave all my money for it.  |
| Use of numbers without quantitative correspondence | I bought a €5 bread and two buns (he only has €5) | It is the price of it (the numbers do not reflect the real price of the buying).  |
| Use of numbers with quantitative correspondence and non arithmetical meaning | Then I give you €2 (this is the price for a bun). | I have bought one bread and it costs €5. UN PA DA: BARA 2€ |
| Arithmetical use of numbers | I bought two €1 buns and a two euros ensaimada; and I have paid four coins (Figure 3). | - |
| Use of numbers with calculator | - | - |

together and spend up to €10 or €15 depending on how many people were playing at the same time. All of the groups used calculators. Three out of four of the groups made receipts and/or shopping lists. In this final instance of the game, Ona said: 'All of this cost us €13 and drew three ensaimadas at €2, one loaf of bread at €5, and two buns at €1.' With this drawing (see Figure 4), she also said: 'I gave €5 and I got two back, so I paid three. We used the calculator to know how much it all cost the three of us.' This is a complete drawing that, without forgetting the objects that had been bought, makes a priority of the money, the calculations and the process of exchanging money.

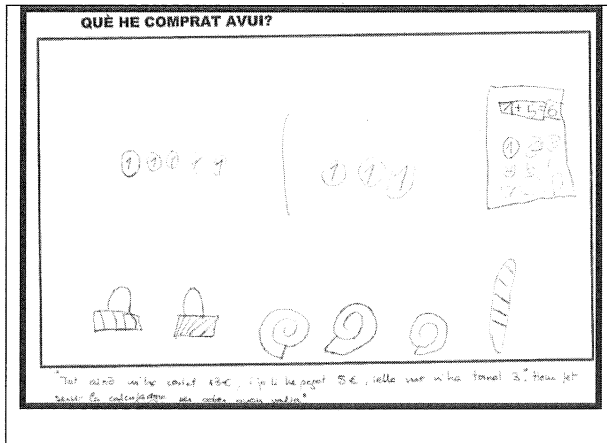


Figure 4. Ona's representation in the third game.

Table 4 illustrates findings concerning the pupils' actions during the third game. The comparison of data from Tables 2, 3 and 4 shows the increasing complexity of mathematical contents in the classroom pupils' discourse as a group, while the comparison of data from Figures 2, 3 and 4 shows the increasing complexity of one pupil's mathematical thinking. Despite the specific learning characteristics of each pupil during her/his engagement in the whole didactical situation, we consider Ona as a representative case of the evolution of the children's mathematical thinking in her class.

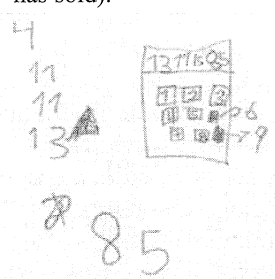
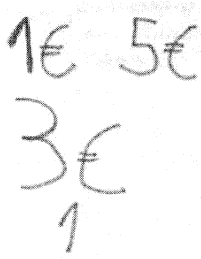
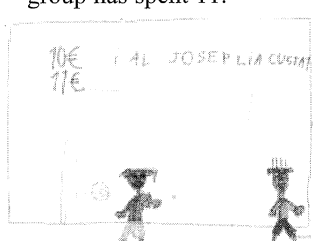
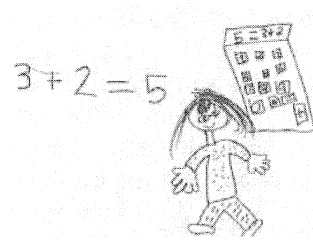
Conclusions

We have presented a symbolic play activity situation that can be considered valuable for the improvement of the children's mathematical thinking. In the initial conversations, none of the pupils mentioned money and arithmetical contents related to the use of money when asked about what we need to buy in a shop. In the final individual interviews, however, 22 of the children made a coherent explanation centred on the word 'money', three gave a well-presented argument centred on the idea of notes and euros, and one gave an incoherent answer. During the classroom activities, they used arithmetical contents and mathematical symbols, mainly numbers, in the different symbolic play situations. When they were prompted to reflect on the meanings and the use of certain mathematical symbols, it was proved their understanding of complex numbers, operations and arithmetical situations such as the one given by spending an amount of money lower or equal, but never higher, than an established quantity.

Generally speaking and from the perspective of the mathematical content worked on during the didactical situation, the pupils in the group:

- used vote counts and comparisons of quantities to establish an order and make decisions democratically;
- observed and analysed the use of money;
- read highly complex numbers (with decimals), making hypotheses about their value and ordering them;

Table 4. Findings related to Game III.

| Categories | Pupils' narratives | |
|--|---|--|
| | Oral narratives and actions | Written narratives |
| Quantitative use of non-numerical words | We put together all our money and we spent it all (3 times 5). | - |
| Use of numbers without quantitative correspondence | - | (The boy reproduces numbers that appear in the display but they do not have the correspondence with what he has sold).  |
| Use of numbers with quantitative correspondence and non arithmetical meaning | I have put €5 and you have put five more (they do not add it up). | We have paid €1, €5 and €3.  |
| Arithmetical use of numbers | All the buying has cost eleven (they have bought a €5, a €3 croissant, a €2 ensaimada and a €1 bun) | We have spent €10 and Josep's group has spent 11.  |
| Use of numbers with calculator | We all spent thirteen (two €1 buns, three €2 ensaimadas and one €5 bread). Pau and Anna gave five each. I gave five and got two back, so I paid three. We used the calculator to know how much it all cost to the three of us (Figure 4). | An ensaimada and a croissant are five euros.  |

- agreed to values for the objects and perform the previously observed actions;
- made use of a calculator and made an approach to adding up;
- read and wrote many numbers, always in context and for some kind of purpose, and worked on writing down the operations; and
- enjoyed using and learning about mathematics in rich situations of guided play.

The data offers relevant conclusions concerning the development of the pupils' actions during the symbolic game. It shows the progressive incorporation of the elements that were initially introduced by visiting a bakery and by the teacher. We have illustrated the case of Ona and the development in her graphic representations. At first, Ona only showed the objects that she had bought. In her second picture she added the representation of money, and finally, in her third picture, she showed the calculations she had made as well as the exchange of money. The evolution in this pupil's mathematical thinking relates the simulated context to numerical and arithmetical situations. In general, the data points to an evolution from the quantitative use of non-numerical words to the arithmetical use of numbers, sometimes being used the calculator. When looking at the pupils' drawings and explanations, we see that at a first stage the numbers are hardly used. At later stages in the second and the third games, the numbers appear with different functions. They do not represent quantities in the simulated context but have a role in the symbolic play: they represent quantities related to prices and coins. However, these objects are not grouped: they represent quantities that need to be grouped, by counting on or with the calculator, especially in the third game; etc.

The process of creating the environment for the symbolic game in the classroom has promoted dynamics involving an exchange of perspectives, the making of joint decisions and the construction of shared meanings. If we compare the results from the first and the third game, we see that the co-operation among pupils turns into a more complex use of the mathematics: they put together the individual quantities of money and make a common list; they add up the total prices more frequently; they remind each other of the calculator; they tend to check the others' use of the money; etc. The social interaction in the classroom simulated context permits each pupil's knowledge about buying and selling to become a public more 'objective' knowledge. The pupils' individual actions facilitate the development of the symbolic play; when working in small groups and making decisions together, the pupils need to achieve an agreement about the use of numbers in a concrete situation. The need for an agreement leads to develop a more complex mathematics that can be seen as the 'objective' knowledge that Perret-Clermont and her colleagues (2004) mention.

As for the use of the simulated context, data shows how the pupils transfer what they have learned in the real-life context (e.g., the visit to the bakery and the conversation with their families) to the simulated classroom context. The visit to the bakery, for example, provided the pupils with a learning experience involving the moments when objects are exchanged for money, the need to delimit prices and make calculations (with or without a calculator), the social function that a paper called a receipt plays, etc. Through the conversation with their families, each pupil investigates the real prices of four products and, later on, shares this knowledge with their colleagues, and prices are compared. Our study made no plans to collect data on the transfer of learning in the simulated context to the real-life context; future research should address this issue.

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