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To cite this article: Axel Masó-Puigdellosas *et al* *J. Stat. Mech.* (2019) 033201

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PAPER: Classical statistical mechanics, equilibrium and non-equilibrium

Stochastic movement subject to a reset-and-residence mechanism: transport properties and first arrival statistics

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Received 31 October 2018

Accepted for publication 28 January 2019

Published 4 March 2019

Online at stacks.iop.org/JSTAT/2019/033201
<https://doi.org/10.1088/1742-5468/ab02f3>



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Abstract. In this work, we consider stochastic movement with random resets to the origin followed by a random residence time before the motion starts again. First, we study the transport properties of the walker, i.e. we derive an expression for the mean square displacement of the overall process and study its dependence on the statistical properties of the resets and the residence times probability density functions (PDFs) and the type of movement. From this general formula, we see that the inclusion of the residence after the resets is able to induce super-diffusive to sub-diffusive (or diffusive) regimes and it can also make a sub-diffusive walker reach a constant mean square displacement or even collapse. Second, we study how the reset-and-residence mechanism affects the survival probability of different search processes to a given position, showing that the long time behavior of the reset and residence time PDFs determine the existence of the mean first arrival time.

Keywords: stochastic particle dynamics, stochastic processes, stationary states, Brownian motion

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1. Introduction

The territorial dynamics of animals are compound multi-stage processes. When they are seeking for food, their behaviour is completely different than when they are resting or socializing around their nest or just exploring potential areas to migrate. Therefore, a complete model to describe spatial occupation of animals should consider a combination of different states and the overall process would end up determining their region of influence, which is technically called the home range [1].

Among these states, the movement for searching has been the main object of study [2–5] but resting phases [6] or migrations [7] have also been analysed. Also, data-oriented models [8] and macroscopic models based in the Fokker–Planck equation [9, 10] have been employed to study the home range of animals in practical cases. Nevertheless, a mesoscopic multi-stage formulation is still lacking.

The inclusion of resets in diffusive motion [11] is a first step in the formulation of a model able to capture the global dynamics from the multiple internal states of the animals. After that, multiple works have been devoted to study many types of processes with different resetting mechanisms [12–32], some of them focusing on the completion time of the processes [33, 34], or using resets to concatenate different processes [35, 36].

Generally, resets have been treated as an instantaneous action that connects two different realisations of a given process. This, for most of the applications of stochastic search (including movement ecology), is not realistic. Nevertheless, some works have introduced a penalizing period after a reset happens using the Michaelis–Mentens reaction scheme. In particular, search processes with constant rate restarts and finite time overheads were studied in [37, 38] and an equation for the first completion time

distribution of a process with constant rate restarts and a general PDF for the time overheads was derived in [39]. Also, in [40] a stochastic process with Poissonian resets followed by a generally distributed quiescent time has been recently studied using a renewal approach.

Following this line, we study the transport properties and the first arrival statistics of a walker which may experience resets after which we introduce a residence (or resting) quiescent period before starting anew. Unlike the previous works, our most interesting results correspond to the particular case where both the reset and the residence time PDFs have a power-law decay. For this case we show that the inclusion of the residence period generates a wide variety of transport regimes. Previous models have already shown this capacity of resting states to modify the transport regime of a process (see for instance [41, 42] or [43] for a successful application).

In an ecological context, the residence period can be interpreted as the time spent by an animal at the nest between two consecutive foraging trips. Furthermore, assuming that the animal ceases to do any task when it decides to return to the nest, the quiescent time could also include the time spent by the animal to go there. This partially solves the unphysical instantaneous nature of resets.

This paper is organised as follows. In section 2 we present the model and we derive an expression for the probability of the walker being at position x at time t . In section 3 we find a formula for the overall mean square displacement (MSD) in terms of the MSD of the movement process and its long-term behaviour is analysed, while in section 4 we study the case where a stationary state is reached. Finally, in section 5 the first arrival statistics of the process are studied and, in the cases where it is finite, a general expression for the mean first arrival time (MFAT) is found. We conclude the work in section 6.

2. The process

Let us introduce the general formulation of the process studied in this paper. In order to describe both the movement process and the resting of the walker in the nest, we define two different states. The first state, $i = 1$, corresponds to the movement stage, while in the second state, $i = 2$, the walker rests at the origin. First, we study the transition probabilities between the two states. Let $\varphi_R(t)$ be the reset time PDF (i.e. the distribution of times the walker spends travelling) and $\varphi_S(t)$ or residence time PDF (i.e. the distribution of times that the walker stays at the origin before moving again). Then, the probability of arriving at state $i = 1, 2$ at time t can be written as

$$j_1(t) = \delta(t) + \int_0^t j_2(t-t')\varphi_S(t')dt' \tag{2.1}$$

$$j_2(t) = \int_0^t j_1(t-t')\varphi_R(t')dt', \tag{2.2}$$

where the $\delta(t)$ in the first equation indicates that the process starts at state $i = 1$. The second term in the first equation and the second equation are the probabilities of

reaching the state i from state i' ; which is the probability of reaching the state i' at any past time $t - t'$ and stay there for a time t' , when the walker switches back to i . Transforming equations (2.1) and (2.2) by Laplace for the time variable we get

$$\hat{j}_1(s) = \frac{1}{1 - \hat{\varphi}_R(s)\hat{\varphi}_S(s)} \tag{2.3}$$

$$\hat{j}_2(s) = \frac{\hat{\varphi}_R(s)}{1 - \hat{\varphi}_R(s)\hat{\varphi}_S(s)} \tag{2.4}$$

where $\mathcal{L}[f(t)] = \hat{f}(s) = \int_0^\infty e^{-st} f(t) dt$ is the laplace transform of $f(t)$. The next step is to introduce the spatial motion in the model. The overall probability that the walker is at point x at time t can be split into two parts $\rho(x, t) = \rho_1(x, t) + \rho_2(x, t)$, where $\rho_i(x, t)$ is the partial probability when it is at state i . For the state $i = 1$ we define the propagator $P(x, t)$ as the time-dependent distribution of the walker position during a single movement stage. On the other hand, we take the position of the the walker to be fixed at $x = 0$ when it is in the resting state ($i = 2$). With these considerations, the time-dependent PDF for each of the states becomes

$$\rho_1(x, t) = \int_0^t dt' j_1(t - t') \varphi_R^*(t') P(x, t'), \tag{2.5}$$

$$\rho_2(x, t) = \delta(x) \int_0^t dt' j_2(t - t') \varphi_S^*(t') \tag{2.6}$$

where $\varphi_{R,S}^*(t) = \int_t^\infty \varphi_{R,S}(t') dt'$. These equations can be read as follows: the position distribution of the walker in each of the states $i = 1, 2$ at time t is the probability of getting there any time before ($j_1(t - t')$ and $j_2(t - t')$, respectively), stay there for the remaining time ($\varphi_R^*(t')$ and $\varphi_S^*(t')$ respectively), with the dynamics of the walker described by the corresponding propagator ($P(x, t')$ and $\delta(x)$, respectively). Transforming by Laplace in time, and substituting the transition probabilities by their explicit expressions given in equations (2.3) and (2.4) we get

$$\hat{\rho}_1(x, s) = \frac{\mathcal{L}[\varphi_R^*(t)P(x, t)]}{1 - \hat{\varphi}_R(s)\hat{\varphi}_S(s)} \tag{2.7}$$

$$\hat{\rho}_2(x, s) = \frac{\hat{\varphi}_S^*(s)\hat{\varphi}_R(s)}{1 - \hat{\varphi}_R(s)\hat{\varphi}_S(s)} \delta(x). \tag{2.8}$$

Hence,

$$\hat{\rho}(x, s) = \frac{\hat{\varphi}_S^*(s)\hat{\varphi}_R(s)\delta(x) + \mathcal{L}[\varphi_R^*(t)P(x, t)]}{1 - \hat{\varphi}_R(s)\hat{\varphi}_S(s)} \tag{2.9}$$

is the PDF of the overall process. In particular, when the time PDF at the origin is taken as $\varphi_S(t) = \delta(t)$, this expression reduces to the case in [44]. Also, if we consider only exponentially distributed resets, this expression reduces to the first result in [40]. There, the case where the reset time and the residence period (refractory period therein)

Stochastic movement subject to a reset-and-residence mechanism: transport properties and first arrival statistics are correlated is also considered and a general propagator for the resulting process is derived.

3. Transport regime analysis

In the previous section we have derived an expression for the probability $\rho(x, t)$ that the walker is at point x at time t . Here we will study the asymptotic behaviour of the MSD. Particularly, we study how the tails of the reset and residence time PDFs modify the transport properties of the overall process, leaving for the next section the study of the cases where the walker reaches a stationary state.

To do so, we start from equation (2.9). If we multiply at both sides by x^2 and integrate over the whole spatial domain we get an equation for the overall MSD in the Laplace space in terms of the time PDFs and the movement MSD:

$$\mathcal{L}[\langle x^2(t) \rangle] = \frac{\mathcal{L}[\varphi_R^*(t)\langle x^2(t) \rangle_m]}{1 - \hat{\varphi}_R(s)\hat{\varphi}_S(s)}. \quad (3.1)$$

Here, the subindex m in $\langle x^2(t) \rangle_m$ indicates that the MSD corresponds to the movement stage, while the expression without subindex $\langle x^2(t) \rangle$ corresponds to the overall process. In order to study more explicitly the MSD we choose a particular expression for the time PDFs:

$$\varphi_R(t) = \frac{t^{\gamma_R-1}}{\tau_m^{\gamma_R}} E_{\gamma_R, \gamma_R} \left[- \left(\frac{t}{\tau_m} \right)^{\gamma_R} \right] \quad (3.2)$$

$$\varphi_S(t) = \frac{t^{\gamma_S-1}}{\tau_s^{\gamma_S}} E_{\gamma_S, \gamma_S} \left[- \left(\frac{t}{\tau_s} \right)^{\gamma_S} \right] \quad (3.3)$$

with

$$E_{\alpha, \beta}(z) = \sum_{n=0}^{\infty} \frac{(-z)^n}{\Gamma(\alpha n + \beta)}, \quad (3.4)$$

being the Mittag-Leffler function, and $0 < \gamma_i \leq 1$ and τ_i (for $i = R, S$) are characteristic parameters. In the large time limit and for $\gamma_i < 1$ we have that $\varphi_i(t) \sim t^{-1-\gamma_i}$. The choice of these PDFs is motivated by the fact that we can recover exponential PDFs for $\gamma_R = 1$ and we can also study power-law behaviours for $\gamma_R < 1$. The first correspond to constant rate events, i.e. at any time there is the same probability of switching from one state to the other. As for the latter, they are a type of PDFs with infinite mean frequently used in the modelling of movement processes. Particularly, sub-diffusion processes are usually modelled by introducing power law tails in the waiting time PDF between two consecutive jumps (see [5]). The difference here is that the resetting mechanism constantly forces the system to wait at the origin, which makes the effect of the long tailed waiting time PDF at the origin much stronger.

Let us also assume that the movement process follows a general MSD which scales as

$$\langle x^2(t) \rangle_m \sim t^p, \quad (3.5)$$

with $0 < p < 2$. Then, the asymptotic behaviour of the overall process MSD can be found by taking the small s limit of equation (3.1) and performing the inverse Laplace transform on the result. Doing so for the whole range of parameters, one finds three different behaviours $\langle x^2(t) \rangle \sim t^a$ depending on the relative values of γ_R and γ_S :

$$a = \begin{cases} \gamma_S - 1, & \gamma_S \leq \gamma_R = 1 \\ p + \gamma_S - \gamma_R, & \gamma_S < \gamma_R < 1 \\ p, & \gamma_S \geq \gamma_R < 1. \end{cases} \quad (3.6)$$

This expression, which groups the main results of this section, shows that the inclusion of the residence at the origin generates a competition between the tails of the PDFs $\varphi_R(t)$ and $\varphi_S(t)$. This produces a rich diversity of transport regimes for the overall process in terms of the exponent of the MSD for the movement process and the reset-and-residence mechanism. In the following we study the different cases of equation (3.6) in detail.

3.1. Long tail $\varphi_R(t)$ PDF

Let us start with the cases for which the reset time PDF has a long tail ($\gamma_R < 1$). In figure 1 the exponent of the MSD at the asymptotic limit, obtained with Monte Carlo simulations, is shown for the whole range $0 < \gamma_S < 1$ and $0 < \gamma_R < 1$, for both a sub-diffusive and a super-diffusive case. There, we can observe the two bottom cases in equation (3.6) separated by the analytical limiting case $\gamma_R = \gamma_S$ (red line). Below the red line, the asymptotic exponent of the MSD does not depend on the values of γ_R and γ_S while above the red line the asymptotic exponent decreases like $\gamma_R - \gamma_S$, as we have found in the second case of equation (3.6).

Interestingly, shape of the regimes in equation (3.6) does not depend on the movement transport regime exponent p . Therefore, the competition is exclusively between the tails of the reset and residence time PDFs. When the tail of $\varphi_R(t)$ decays equally or slower than the tail of $\varphi_S(t)$ ($\gamma_S \geq \gamma_R < 1$), asymptotically there are less walkers resetting their position than leaving the origin, which makes the residence stage negligible. On the other hand, when the tail of $\varphi_S(t)$ decays slower than the tail of $\varphi_R(t)$ ($\gamma_S < \gamma_R < 1$), the residence PDF becomes significant and the overall transport regime is affected by the reset and residence time PDFs. In fact, this change of behaviour is a consequence of the requirement of the walker to have restarted its position for the residence mechanism to be triggered. Therefore, when the reset times are asymptotically longer than the residence times, the asymptotic behaviour of the MSD is as if there was no residence [44]. Otherwise, when the residence times are asymptotically longer, the effect of the particles stacked at the origin becomes qualitatively relevant.

3.2. Exponential $\varphi_R(t)$ PDF

When the reset time PDF $\varphi_R(t)$ is exponential, the asymptotic behaviour of the MSD changes drastically. This case includes both power law and exponential PDFs for $\varphi_S(t)$. In figure 2 we show the simulated MSD for long tailed residence time PDFs and different movement: sub-diffusive and diffusive processes, and super-diffusive Lévy walks. In all three cases, the MSD increases for short times until it reaches a maximum value and

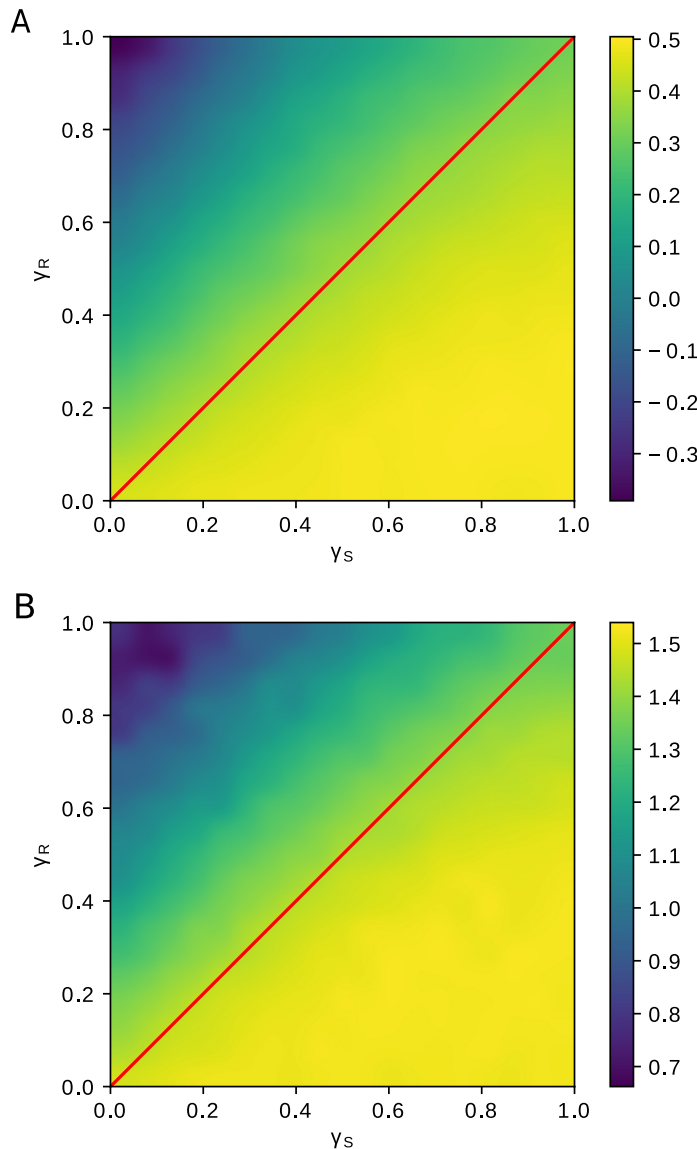


Figure 1. Exponent a of the asymptotic behaviour of the MSD of the overall process $\langle x^2(t) \rangle \sim t^a$ with respect to the long tailed reset and residence time PDFs exponents γ_R and γ_S respectively is shown. The plots correspond to simulations of two different movement processes: a sub-diffusive process with $p = 0.5$ in (A) and a super-diffusive Lévy walk with $p = 1.5$ in (B). For both cases, in the region below the red line ($\gamma_R = \gamma_S$) the exponent is constant and equal to the movement MSD exponent $a = p$ while in the region above the line the exponent diminishes progressively.

starts to decrease as predicted with the first case in equation (3.6). The asymptotic decay depends exclusively on the residence time PDF exponent γ_S and in this regime the movement process only modifies multiplicatively the overall MSD.

To summarize, in figure 3 we present a phase diagram with the possible transport regimes. There we can see that a super-diffusive process ($p > 1$) can be transformed to a sub-diffusive (or diffusive) process when the reset-and-residence mechanism is taken into account. Similarly, a sub-diffusive process ($p < 1$) can derive into a transport

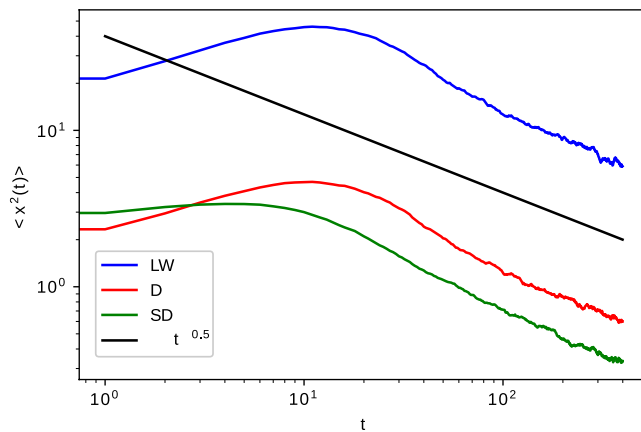


Figure 2. The time evolution of the MSD of a simulation of an overall process with $\gamma_R = 1$ and $\gamma_S = 0.5$ is plotted for a sub-diffusive (SD), a diffusive (D) and a Lévy walk (LW) super-diffusive motion. In all three cases the MSD decreases as $t^{-0.5}$ in the long time limit as predicted by the first case in equation (3.6). See the guide line (black line in the plot) for comparison.

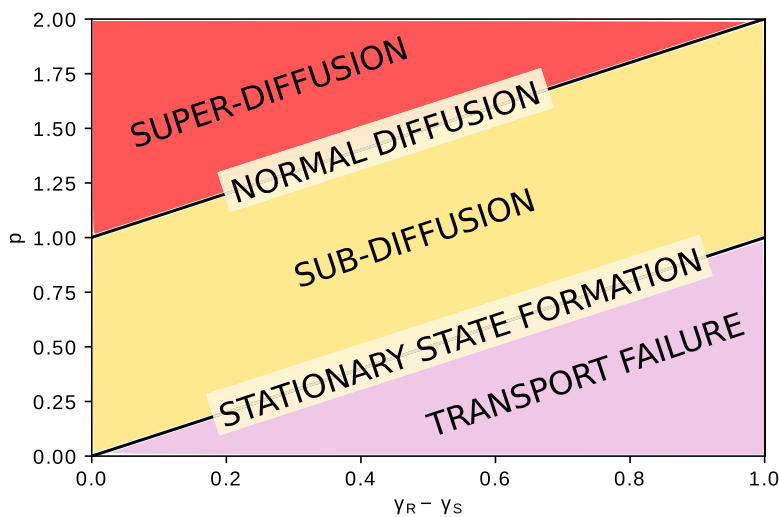


Figure 3. Schematic plot of the different overall transport regimes in terms of the transport regime of the movement process p and the difference between the decaying exponents of the reset and residence time PDFs $\gamma_R - \gamma_S$. In the red region the overall behaviour is super-diffusive. In the yellow region it is sub-diffusive and the pink represents the values for which we have transport failure.

failure regime. Therefore, the reset-and-residence mechanism can be also seen as a tool to slow down the transport regime.

4. Stationary state

In the previous section we have derived in a general manner the transport regime of the overall process and we have also studied how the reset-and-residence mechanism

Stochastic movement subject to a reset-and-residence mechanism: transport properties and first arrival statistics affects the long term behaviour of the system. In this section we study the special case where both the residence and the reset time PDFs are exponential ($\gamma_S = \gamma_R = 1$), so a stationary state is reached. Concretely, we derive a formula for the overall MSD in terms of the MSD of the movement and we find the stationary distribution for some well-known movement processes.

To do so, we start from equation (3.1) and the time PDFs from equations (3.2) and (3.3) with $\gamma_R = 1$ and $\gamma_S = 1$ respectively. With these parameters the Laplace transform of the overall MSD reads

$$\mathcal{L}[\langle x^2(t) \rangle] = \frac{\mathcal{L}[e^{-\frac{t}{\tau_m}} \langle x^2(t) \rangle_m]}{1 - \frac{1}{(1+\tau_m s)(1+\tau_s s)}}. \tag{4.1}$$

In order to find the asymptotic behaviour of the MSD we make use of the final value theorem $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s \mathcal{L}[f(t)]$. Doing so, the stationary MSD can be easily found from equation (4.1) giving

$$\langle x^2 \rangle_{st} = \frac{1}{\tau_m + \tau_s} \int_0^\infty e^{-\frac{t}{\tau_m}} \langle x^2(t) \rangle_m dt. \tag{4.2}$$

For a motion with $\langle x^2(t) \rangle_m = 2Dt^p$ with D, p positive constants, the stationary MSD takes the simple form

$$\langle x^2 \rangle_{st} = 2D \Gamma(p + 1) \frac{\tau_m^{p+1}}{\tau_m + \tau_s}. \tag{4.3}$$

This result has been compared with Monte Carlo simulations in figure 4 for three different types of movement. Let us now calculate the stationary distribution. Doing so it is easy to see that the stationary distribution can be written in terms of the propagator $P(x, t)$ as

$$\rho_{st}(x) = \frac{\tau_s}{\tau_m + \tau_s} \delta(x) + \frac{\hat{P}(x, s = \frac{1}{\tau_m})}{\tau_m + \tau_s}, \tag{4.4}$$

which has been recently found in [40]. For a propagator of the form

$$P^{SD}(k, s) = \frac{1}{s + D_\gamma s^{1-\gamma} k^2}$$

in the Fourier–Laplace space, which corresponds to a sub-diffusive process for $\gamma < 1$ and diffusive for $\gamma = 1$, the stationary distribution is

$$\rho_{st}^{SD}(x) = \frac{\tau_s}{\tau_m + \tau_s} \delta(x) + \frac{\tau_m}{\tau_m + \tau_s} \frac{e^{-\frac{|x|}{\sqrt{D_\gamma \tau_m}}}}{\sqrt{4D_\gamma \tau_m^\gamma}}. \tag{4.5}$$

If instead of a sub-diffusive (or diffusive) motion we consider a Lévy flight, its propagator is

$$P^{LF}(k, s) = \frac{1}{s + D_\alpha |k|^\alpha}$$

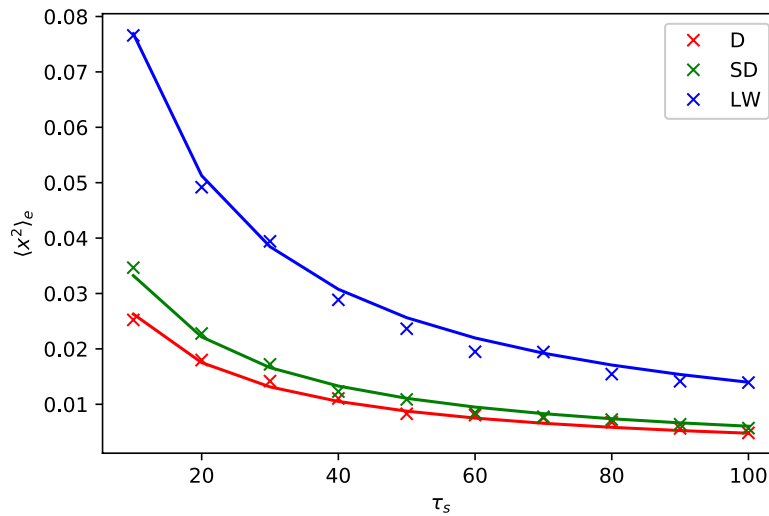


Figure 4. Stationary MSD for a sub-diffusive (SD), a diffusive (D) and a Lévy flight (LF) propagator when they are subject to reset times which are exponential distributed with $\tau_m = 10$ and retained at the origin during a time given by an exponential PDF with different means τ_s . For the propagators we have chosen $p = 0.5$, $D = 1.05 \cdot 10^{-2}$ for the sub-diffusive, $p = 1$, $D = 2.63 \cdot 10^{-3}$ for the diffusive and $p = 1.1$, $D = 6.11 \cdot 10^{-3}$ for the Lévy walk. The crosses correspond to the simulated distributions while the solid curves show the analytical result given in equation (4.3).

in the Fourier–Laplace space, where $\alpha < 2$ and D_α is the corresponding diffusion constant. The stationary distribution reads

$$\rho_{\text{st}}^{\text{LF}}(x) = \frac{\tau_s}{\tau_m + \tau_s} \delta(x) + 2 \frac{\tau_m}{\tau_m + \tau_s} \int_0^\infty \frac{\cos(kx)}{1 + \tau_m D_\alpha k^\alpha} dk. \tag{4.6}$$

This distribution has divergent second order moment since it decays as $\frac{1}{|x|^{1+\alpha}}$ for large $|x|$. Therefore, despite a stationary state is reached the average region of space covered by the walker around the nest is infinite. Contrarily, for the sub-diffusive and diffusive movement processes we have a finite stationary MSD, the value of which is given by equation (4.3) with properly chosen parameters.

5. Mean first arrival time

In the previous section we have analysed the long-term behaviour of the overall process. Here we perform a first arrival time analysis which is still lacking despite of the previous works including a quiescent period after the resets. It consists on determining the conditions for which the MFAT is finite, taking into account finite and infinite mean PDFs for the reset and residence time PDFs. To do so, we build up a mesoscopic equation for the overall survival probability $\sigma_x(t)$, which is the probability of the walker not having arrived to x at time t . Let us separate the overall survival probability into three temporal non-overlapping processes. If the walker starts at the origin but is moving (i.e. at the state $i = 1$), the three contributions to the overall survival probability are:

- (i) The walker has not yet restarted at time t . This happens with probability $\varphi_R^*(t)$ and in this case the overall survival probability is the same as the motion survival probability $Q_x(t)$.
- (ii) The walker has restarted its position once at time $t' < t$, during which it has not arrived to x , and during the period (t', t) it has not left the origin. The first happens with probability $\varphi_R(t')Q_x(t')$, the second with probability $\varphi_S^*(t - t')$ with $0 \leq t' \leq t$.
- (iii) The walker has restarted its position at least once at time $t' < t$ and it has not reached x before t' , which happens with probability $\varphi_R(t')Q_x(t')$ as in the previous case. However, here the walker leaves the origin after a time $t'' < t - t'$ and in this case the walker is exactly at the initial scenario but instead of having a time t , we have to subtract the time of the first non-triggering journey ($t \rightarrow t - t' - t''$). This happens with probability $\varphi_S(t'')\sigma_x(t - t' - t'')$ given that the first reset was at time t' . These scenarios must be taken into account for all $t'' < t - t'$ and $t' < t$.

Putting the three contributions together we get:

$$\begin{aligned} \sigma_x(t) = & \varphi_R^*(t)Q_x(t) + \int_0^t \varphi_R(t')Q_x(t')\varphi_S^*(t - t')dt' \\ & + \int_0^t \varphi_R(t')Q_x(t')dt' \int_0^{t-t'} \varphi_S(t'')\sigma_x(t - t' - t'')dt'', \end{aligned} \tag{5.1}$$

where $Q_x(t)$ is the survival probability of the movement process and the first, second and third terms in the rhs correspond to cases (i), (ii) and (iii), respectively. Applying the Laplace transform to equation (5.1) we get

$$\hat{\sigma}_x(s) = \frac{\mathcal{L}[\varphi_R^*(t)Q_x(t)] + \mathcal{L}[\varphi_R(t)Q_x(t)]\hat{\varphi}_S^*(s)}{1 - \mathcal{L}[\varphi_R(t)Q_x(t)]\hat{\varphi}_S(s)}, \tag{5.2}$$

which has been also found as a particular case in [40] by assuming non-correlated reset and residence time PDFs on their general result. This equation is completely general so it is valid for any survival probability of the movement process $Q_x(t)$ and any reset and residence time PDFs $\varphi_R(t)$ and $\varphi_S(t)$, respectively. Here we focus on studying the asymptotic behaviour of the overall survival probability for $Q_x(t) \sim t^{-q}$ with $0 < q < 1$. This choice encompasses some processes associated with animal foraging as shown in the particular examples below. For $\varphi_R(t)$ and $\varphi_S(t)$ we consider again expressions (3.2) and (3.3), respectively.

The existence of a MFAT can be determined by taking the limit $s \rightarrow 0$ in equation (5.2) ($T_F = \lim_{s \rightarrow 0} \hat{\sigma}(s)$). In this limit, corresponding to the long t limit, the dominant term in the numerator of equation (5.2) depends on the tail exponents of the PDFs $\varphi_R(t)$ and $\varphi_S(t)$. Concretely, we can distinguish three different cases:

- (a) $\gamma_S \geq \gamma_R + q$. At long times, the first term in the numerator of equation (5.2) dominates over the second (or they are equal in the limiting case). The MFAT is infinite since the survival probability decays as

$$\sigma_x(t) \sim t^{\gamma_R+q-1}. \tag{5.3}$$

- (b) $\gamma_S < \gamma_R + q$, $\gamma_S < 1$. At long times, the second term in the numerator of equation (5.2) is the dominant term. Here, the MFAT is infinite again, with a survival probability decaying as

$$\sigma_x(t) \sim t^{\gamma_S-1}. \tag{5.4}$$

- (c) $\gamma_S < \gamma_R + q$, $\gamma_S = 1$. In this scenario, the MFAT is finite and it can be written as

$$T_F = \frac{I_1 + \tau_s I_2}{1 - I_2}, \tag{5.5}$$

with

$$I_1 = \int_0^\infty E_{\gamma_R,1} \left[- \left(\frac{t}{\tau_m} \right)^{\gamma_R} \right] Q_x(t) dt \tag{5.6}$$

and

$$I_2 = \int_0^\infty \frac{t^{\gamma_R-1}}{\tau_m^{\gamma_R}} E_{\gamma_R,\gamma_R} \left[- \left(\frac{t}{\tau_m} \right)^{\gamma_R} \right] Q_x(t) dt, \tag{5.7}$$

where we have made use of equations (3.2) and (3.3).

From this we see that, when studying the overall survival probability, the reset time PDF couples to the survival probability of the movement process and is the joint tail the one which competes with the residence time PDF. Therefore, the type of movement has a direct effect on which distribution determines the tail exponent of the overall survival probability. To illustrate this, we next study these general results for some particular cases.

5.1. Infinite MFAT

Let us leave the case (c) in the previous Section apart for the moment. From cases (a) and (b) we see that there are two different regions in the state space where the governing tail differs, limited by the plane $\gamma_S = \gamma_R + q$. In the following we test this for two different movement processes: a sub-diffusive random walk (figure 5(A)) and a Lévy flight (figure 5(B)).

The survival probability of a sub-diffusive jump process has been studied in [45], where it was found to be

$$Q_x^{SD}(t) = \frac{x}{\sqrt{Dt}^\gamma} \sum_{k=0}^\infty \frac{\left(-\frac{x}{\sqrt{Dt}^\gamma} \right)^k}{(k+1)! \Gamma\left(1 - \frac{\gamma}{2} - k\frac{\gamma}{2}\right)} \tag{5.8}$$

with $0 < \gamma < 1$. This, in the large time limit decays as

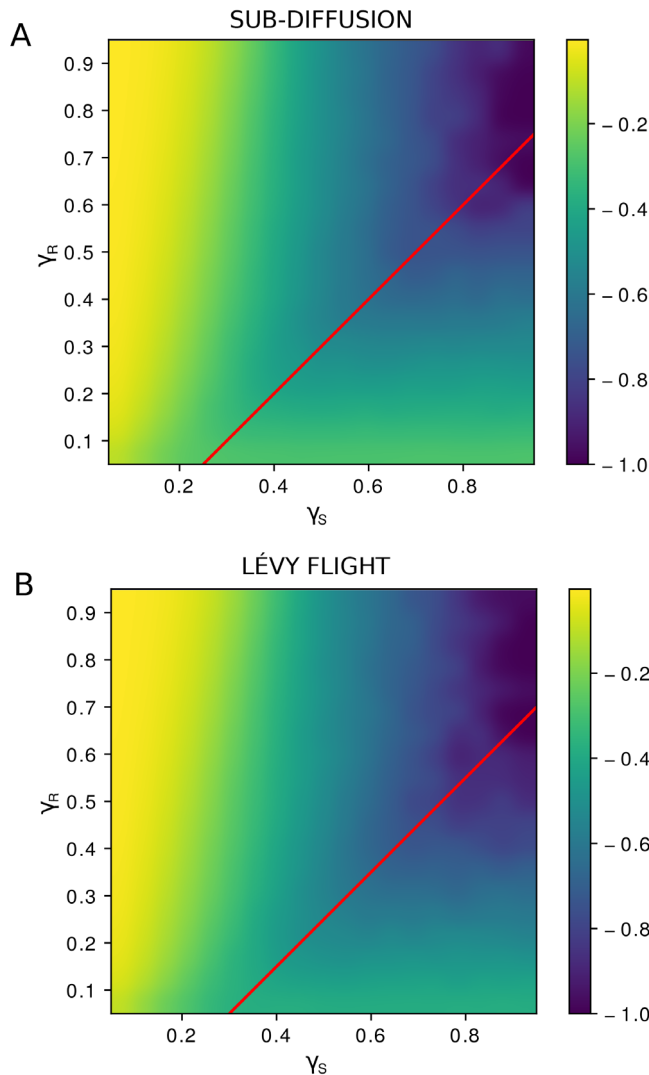


Figure 5. Exponent b of the asymptotic behaviour of the survival probability of the overall process $\sigma_x(t) \sim t^{-b}$ with long tailed reset and residence time PDFs is shown. Their exponents are γ_R and γ_S and the plots are for simulations of two different movement processes: a sub-diffusive process with $\gamma = 0.5$ in (A) and a super-diffusive Lévy walk with $\alpha = 1.5$ in (B). For both cases, in the region above the red line ($\gamma_S = \gamma_R + q$ with $q = 0.25$ in (A) and $q = 0.33$ in (B)) b only changes horizontally with γ_S while below the line b only varies vertically with γ_R . These are the results predicted by the cases (a) and (b) in the main text respectively.

$$Q_x^{\text{SD}} \sim t^{-\frac{\gamma}{2}}. \tag{5.9}$$

For $\gamma = 1$, we can recover the well-known decay for the survival probability of a diffusive process. From the asymptotic expression we can identify $q = \frac{\gamma}{2}$ and, therefore, in this case the limiting plane is $\gamma_S = \gamma_R + \frac{\gamma}{2}$, which is illustrated as a red line in figure 5(A) for the particular case $\gamma = 0.5$. There we can see that, below the line, the tail exponent of the overall survival probability increases upwards and remains constant in the horizontal direction, which means that it depends on the reset tail only.

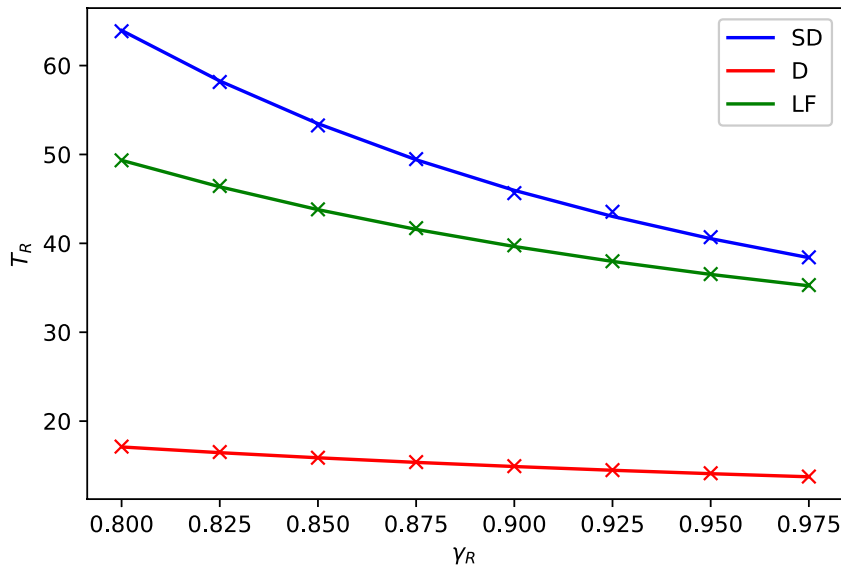


Figure 6. The simulated MFAT (crosses) with an exponential residence time PDF with mean $\tau_s = 10$ and for three representative cases of subdiffusion (SD) with $\gamma = 0.5$, diffusion (D) with $\gamma = 1$ and a Lévy flight with $\alpha = 1.5$, all with $D = 0.1$. We compare our simulations to the analytical result in equation (3.4) (solid curves) for different reset time PDF tail exponents γ_R and $\tau_m = 10$. Concretely, the MFAT is computed at a distance $x = 0.5$ from the origin.

Contrarily, above the red line the decaying exponent of the overall survival probability increases from left to right and remains constant in the vertical direction. Therefore, in this case, it only depends on the residence tail, which is in agreement with cases (a) and (b) above. This general behaviour becomes less clear near the red lines, where the tail competition is evener. This can be amended by running considerably longer simulations.

Regarding the Lévy flight process, an expression for the Laplace transform of the first arrival survival probability was derived in [46]:

$$\hat{Q}_x^{\text{LF}}(s) = \frac{\alpha D^{1/\alpha} \sin(\pi/\alpha)}{\pi s^{1/\alpha}} \int_0^\infty \frac{1 - \cos(kx)}{s + Dk^\alpha} dk. \quad (5.10)$$

This scales asymptotically as

$$Q_x^{\text{LF}}(t) \sim t^{\frac{1}{\alpha}-1}. \quad (5.11)$$

Here $1 < \alpha < 2$, being $\alpha = 2$ the diffusive limit. In this case, the limiting surface is $\gamma_S = \gamma_R + \frac{1}{\alpha}$, which has been plotted for the particular value $\alpha = 1.5$ in figure 5(B) as a red straight line. As for the previous example, the line separates two different regions in which the dominant exponent is γ_S above the red line and γ_R below.

5.2. Finite MFAT

From the three different cases from equation (5.2), the (c) scenario is the only one where the MFAT is finite. In this case, equation (5.5) gives us its value for different movement survival probabilities. As for the previous situations, we consider the two cases where

the motion is sub-diffusive and a Lévy flight, the survival probabilities of which are in equations (5.8) and (5.10) respectively. In figure 6 we show the simulated MFAT for a sub-diffusive, a diffusive (as a limit of the first) and a Lévy flight process in the (c) scenario. The simulations are compared with the result obtained by numerically integrating equation (5.5) for the different cases.

6. Conclusions

We have studied how a stochastic residence period at the resetting position affects the overall dynamics of a random walker. More specifically, we have employed a renewal approach to derive analytical expressions for the MSD and the survival probability (or MFAT when it is finite), which have been validated with Monte Carlo simulations of different processes such as diffusion, Lévy walks or Lévy flights.

For the MSD we have found that, on one hand, for long tailed reset and residence time PDFs the MSD of the overall process depends strongly on the relation between the tail exponents of both PDFs (γ_R for resets and γ_S for residence times). Concretely, for $\gamma_R \geq \gamma_S$ the overall MSD exponent is the same as the one for the movement process while for $\gamma_R < \gamma_S$ the exponent of the movement process is diminished by a factor $\gamma_R - \gamma_S$. On the other hand, for exponentially distributed reset times, if the residence time PDF is long tailed the overall process collapses to the origin while for an exponential residence time PDF a stationary MSD is reached (see equation (4.2)).

Regarding the decaying exponent of the survival probability when both time PDFs are long tailed, we have seen that the tail of the residence time PDF competes with both the tail of the movement process survival probability and the tail of the reset time PDF. In this case the MFAT is always infinite. Contrarily, when the residence time PDF is exponential, a finite MFAT may be found but only when $q + \gamma_R > 1$ with q and γ_R being the tail exponents of the survival probability of the movement process and the reset time PDF respectively.

We note that, in the context of movement ecology, resets are a natural mechanism to concatenate different trials of a particular behaviour of the animals. In this work we have tried to include explicitly the period that they spend in the nest separating excursions for feeding, mating or any other basic functions. The next step towards a complete model could be the introduction of multiple search strategies with different movement patterns to be chosen during the resting period, similarly to what has been done in [35, 36] for instance. Also, regarding the interpretation of the quiescent time as the time needed by the animal to return to the nest, a further step to be addressed in future works would be the introduction of a process driving the individual gradually towards the nest instead of leaving it in a motionless state.

Acknowledgments

This research has been supported by the Spanish government through Grant No. CGL2016-78156-C2-2-R.

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