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Efficiency of harvesting energy from colored noise by linear oscillators

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We investigate the performance of a linear electromechanical oscillator as an energy harvester of finite-bandwidth random vibrations. We derive exact analytical expressions for the net electrical power and the efficiency of the conversion of the power supplied by the noise into electrical power for arbitrary colored noise. We apply our results to the important case of exponentially correlated noise and discuss the tuning of parameters to achieve good performance of the device.

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I. INTRODUCTION

The concept of harvesting ambient energy to provide power for small self-contained sensors and actuators, where batteries and other power sources are inconvenient because they need to be replaced or refueled regularly, has attracted considerable interest in recent years. A wide range of energy harvesting systems and applications have been studied; for recent reviews, see, for example, Refs. [1–3]. The most prominent type of systems are mechanical vibration energy harvesters that convert kinetic energy via electromagnetic, electrostatic, or piezoelectric transductions into electrical energy [4–8]. A recent study considers flexoelectric membranes as the electromechanical transduction mechanism [9]. Mechanical vibration harvesters can be modeled as mass-spring systems. Early studies considered linear springs and harmonic oscillators and treated the external vibrations as sinusoidal vibrations. In such systems, the harvester extracts maximum energy from the ambient vibrations, if the excitation frequency matches the natural frequency of the system, known as resonant energy harvesting [4]. Nearly all current vibration transducers operate in this regime [3]. Efficient operation of the vibration harvester then requires appropriate active or passive tuning [5,7,10].

The ambient energy is however rarely concentrated in a narrow band around a dominant excitation frequency and is typically distributed over a broad range of frequencies. To overcome the limitations of resonant energy harvesting, various groups have begun to study mass-spring systems with nonlinear springs and nonlinear oscillators [11–14]. In particular, the power output of unimodal and bimodal Duffing oscillators has been investigated [12,13,15–23]. Broadband ambient vibrations are typically modeled by Gaussian white noise, and the performance of linear resonant and nonlinear energy harvesters has been investigated [17,23–25]. Using the Fokker-Planck equation to describe Duffing-type energy harvesters in a Gaussian white noise environment, Daqaq [17] and Green *et al*. [23] established that the mean power output of the device is not affected by the nonlinearity of the spring. These findings were recently confirmed by Halvorsen [25] who obtained upper bounds on the power out of linear and nonlinear energy harvesters driven by Gaussian white noise. Halvorsen [25] concluded that "nonlinear harvesters are not fundamentally better than linear ones." Nonlinear oscillators may still be advantageous in that the size of the harvesting device can be reduced without affecting the power output [23].

Most studies of vibration energy harvesters have considered either sinusoidal excitations or Gaussian white noise excitations. However, ambient vibrations in applications can deviate from these idealizations [26]. The case of finite-bandwidth random vibrations, i.e., external colored, noise has received very little attention [14,17,26]. References [14,26] consist of experimental and simulation studies. Daqaq [17] employs approximate methods to obtain the power output of a Duffing oscillator driven by Ornstein-Uhlenbeck noise. A comprehensive theory of linear and nonlinear vibration energy harvesters driven by colored noise is lacking. Our aim is to provide rigorous analytical results for the power output and the efficiency of linear harvesters driven by arbitrary colored noise. The paper is organized as follows. In Sec. II we introduce the evolution equations for the vibration energy harvester. We assume transduction via piezoelectricity. We show that the stationary state is stable in the mean. In Sec. III we derive closed-form expressions for the second moments of the state variables of the harvester for arbitrary colored noise. Section IV deals with the power output and the efficiency of the energy harvesting device. We obtain analytical expressions for the power and the efficiency in terms of the correlation function of the colored noise. These results are applied in Sec. V to exponentially correlated random vibrations. We also consider the white noise limit and compare the power and efficiency for the finite-bandwidth case and the broadband case. We provide a summary of our results in Sec. VI.

II. ELECTROMECHANICAL OSCILLATOR

An electromechanical oscillator is a device that converts the power supplied by external noise, usually from an ambient source, into electrical energy (Fig. 1). The conversion is done in two steps. First, the external noise drives a damped oscillator. Second, the oscillator is coupled to a capacitor that stores the electrical energy. The coupling between the first and second steps is done by a transducer mechanism based on piezoelectricity. The equation for the position of the stochastically driven damped oscillator is

$$\ddot{x} + b\dot{x} + c(x,V) + \frac{dU}{dx} = \xi(t), \tag{2.1}$$

where $U(x)$ is the potential energy, b is the damping coefficient, $\xi(t)$ is the random driving force, and $c(x,V)$ is the reaction force due to the motion-to-electricity conversion

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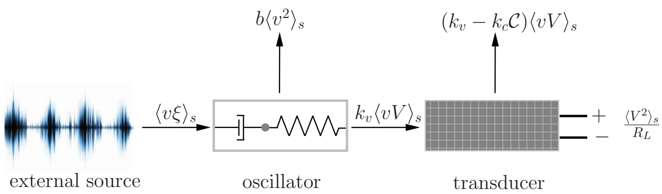

FIG. 1. (Color online) Schematic of the conversion mechanism of the external random power to the net electrical power. The power transferred is shown for each step.

mechanism and the dot stands for temporal derivative. It has the same sign as the dissipative force and opposes the motion. This arises from the energy fraction that is taken from the kinetic energy and converted into electric energy. The simplest expression for this function is $c(x,V) = k_v V$, where $k_v > 0$ is a piezoelectric parameter and $V(t)$ is the voltage. The equation for the dynamics of the voltage has to take into account the capacitor with capacitance $\mathcal{C}$, the load resistance of the piezoelectric component, and the connecting function $F(\dot{x},V)$ with the oscillator:

$$\dot{V} = F(\dot{x},V) - \frac{V}{\tau_p}, \tag{2.2}$$

where $\tau_p = R_L\mathcal{C}$ is the characteristic time of the capacitor's charge process. This time is larger than any other characteristic time of the system. The simplest form of the connecting function $F(\dot{x},V)$ is $k_c\dot{x}$, where k_c is another positive piezoelectric parameter. Finally, the stochastic equations for the electromechanical energy harvester are given by

$$\ddot{x} + b\dot{x} + k_v V + U'(x) = \xi(t), \tag{2.3a}$$

$$\dot{V} = k_c\dot{x} - \frac{V}{\tau_p}. \tag{2.3b}$$

If we assume a harmonic potential $U(x) = \omega_0^2 x^2/2$, take the Laplace transform, and combine both equations in (2.3), we obtain a single equation for the random variable $x(t)$,

$$\ddot{x} + b\int_0^t \dot{x}(t')M(t-t')dt' + \omega_0^2 x = \xi(t) - k_v V_0 e^{-t/\tau_p}, \tag{2.4}$$

where the memory kernel is given by

$$M(t) = \delta(t) + \frac{k_v k_c}{b}e^{-t/\tau_p} \tag{2.5}$$

and $V_0 = V(t=0)$ and is constant. The stochastic equation (2.4) resembles a generalized Langevin equation; however $M(t)$ and $\xi(t)$ do not obey the fluctuation-dissipation theorem.

We expect the electromechanical oscillator to reach a stable stationary state as time goes to infinity. To check this, we write (2.3) as a three-variable first-order system:

$$\frac{dx}{dt} = v, \tag{2.6a}$$

$$\frac{dv}{dt} = -bv - k_v V - \omega^2 x + \xi(t), \tag{2.6b}$$

$$\frac{dV}{dt} = k_c v - \frac{V}{\tau_p}, \tag{2.6c}$$

or in matrix notation

$$\frac{d\boldsymbol{u}}{dt} = A\boldsymbol{u} + \boldsymbol{z}(t), \tag{2.7}$$

where

$$\boldsymbol{u} = \begin{pmatrix} x \\ v \\ V \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 1 & 0 \\ -\omega^2 & -b & -k_v \\ 0 & k_c & -\frac{1}{\tau_p} \end{pmatrix}, \quad \boldsymbol{z}(t) = \begin{pmatrix} 0 \\ \xi(t) \\ 0 \end{pmatrix}. \tag{2.8}$$

Taking the average of (2.6) or (2.7), we obtain

$$\frac{d\boldsymbol{m}}{dt} = A\boldsymbol{m}, \tag{2.9}$$

where $\boldsymbol{m} = \langle \boldsymbol{u}\rangle$. Clearly, in the stationary state the mean $\boldsymbol{m}$ vanishes, $\langle x\rangle = \langle v\rangle = \langle V\rangle = 0$. We apply the the Routh-Hurwitz stability criterion [27] to the system (2.9) and find that all Hurwitz determinants are positive:

$$c_3 = \frac{\omega^2}{\tau_p}, \tag{2.10a}$$

$$\Delta_1 = b + \frac{1}{\tau_p}, \tag{2.10b}$$

$$\Delta_2 = bk_ck_v + b\omega^2 + \frac{b}{\tau_p^2} + \frac{b^2 + k_ck_v}{\tau_p}, \tag{2.10c}$$

$$\Delta_3 = \frac{b\omega^2}{\tau_p^3} + (b^2 + k_ck_v)\frac{\omega^2}{\tau_p^2} + \frac{b\omega^4}{\tau_p}. \tag{2.10d}$$

This implies that the oscillator is stable in the mean and approaches $\boldsymbol{m} = \boldsymbol{0}$ for large times.

III. SECOND MOMENTS

From (2.6) we obtain the following equations for the second moments of the electromechanical oscillator:

$$\frac{d\langle x^2\rangle}{dt} = 2\langle xv\rangle, \tag{3.1}$$

$$\frac{d\langle xv\rangle}{dt} = \langle v^2\rangle + \langle x\xi\rangle - b\langle xv\rangle - k_v\langle xV\rangle - \omega^2\langle x^2\rangle, \tag{3.2}$$

$$\frac{d\langle v^2\rangle}{dt} = -2b\langle v^2\rangle - 2k_v\langle vV\rangle - 2\omega^2\langle xv\rangle + 2\langle v\xi\rangle, \tag{3.3}$$

$$\frac{d\langle xV\rangle}{dt} = \langle vV\rangle + k_c\langle xv\rangle - \frac{1}{\tau_p}\langle xV\rangle, \tag{3.4}$$

$$\frac{d\langle vV\rangle}{dt} = -\left(b + \frac{1}{\tau_p}\right)\langle vV\rangle - k_v\langle V^2\rangle - \omega^2\langle xV\rangle + \langle V\xi\rangle + k_c\langle v^2\rangle, \tag{3.5}$$

$$\frac{d\langle V^2\rangle}{dt} = 2k_c\langle vV\rangle - \frac{2}{\tau_p}\langle V^2\rangle. \tag{3.6}$$

To close the system of equations for the second moments, we need to find the correlations of the electromechanical oscillator state variables with the external noise, $\langle x\xi\rangle$, $\langle v\xi\rangle$, and $\langle V\xi\rangle$. We assume that the noise is stationary with vanishing mean and correlation function $C(s)$:

$$\langle \xi \rangle = 0,\ \langle \xi(t)\xi(t+u)\rangle = C(u). \tag{3.7}$$

We write the solution of (2.4), or (2.6), formally as

$$x(t) = \langle x(t)\rangle + \int_0^t H(t-u)\xi(u)du, \tag{3.8}$$

where $H(t)$ is a Green's function to be determined. Taking the Laplace transforms of (2.4) and (3.8), we obtain

$$\hat{x}(s) = \hat{f}(s) + \hat{H}(s)\hat{\xi}(s). \tag{3.9}$$

Here the hat symbol denotes the Laplace transform with parameter s, $\hat{f}(s)$ is the Laplace transform of $\langle x(t)\rangle$, and

$$\hat{H}(s) = \frac{1}{s^2 + bs\hat{M}(s) + \omega^2}, \tag{3.10a}$$

$$\hat{M}(s) = 1 + \frac{k_v k_c}{b\left(s + \frac{1}{\tau_p}\right)}, \tag{3.10b}$$

$$\hat{f}(s) = \hat{H}(s)\left\{x_0[s + b\hat{M}(s)] + v_0 - \frac{k_v k_c}{s + \frac{1}{\tau_p}}\right\}, \tag{3.10c}$$

where $x_0 = x(t=0)$ and $v_0 = v(t=0)$. Analogously, we can find for the velocity a solution of the form

$$v(t) = \langle v(t)\rangle + \int_0^t G(t-u)\xi(u)du, \tag{3.11}$$

where the Laplace transform of $\langle v(t)\rangle$ is $s\hat{f}(s) - v_0$ and $\hat{G}(s) = s\hat{H}(s)$. Finally, from (3.7), (3.8), and (3.11) we obtain

$$\langle x(t)\xi(t)\rangle = \int_0^t H(u)C(u)du, \tag{3.12}$$

$$\langle v(t)\xi(t)\rangle = \int_0^t G(u)C(u)du. \tag{3.13}$$

From (2.6c) we find that

$$\langle V(t)\xi(t)\rangle = k_c \int_0^t e^{-(t-u)/\tau_p}\langle v(u)\xi(t)\rangle du = k_c \int_0^t C(u)\left[\int_0^u G(z)e^{-(u-z)/\tau_p}dz\right]du. \tag{3.14}$$

IV. ENERGY BALANCE AND EFFICIENCY

The energy of the oscillator is sum of the kinetic and the potential energy, $E = v^2/2 + U(x)$. The energy rate or power of the oscillator can be calculated by taking the time derivative of the energy,

$$\frac{dE}{dt} = v[\ddot{x} + U'(x)]. \tag{4.1}$$

Taking into account (2.3a) and averaging, we obtain the mean power balance equation

$$\frac{d\langle E\rangle}{dt} = -b\langle v^2\rangle - k_v\langle vV\rangle + \langle v\xi\rangle, \tag{4.2}$$

where $\langle v\xi\rangle$ is the power supplied by the noise, $b\langle v^2\rangle$ the power dissipated by friction, and $k_v\langle vV\rangle$ is the power transferred from the oscillator to the transducer. In the steady state, the power transferred from the oscillator to the transducer, $k_v\langle vV\rangle$, can be related to the net power converted into electrical power $k_c\tau_p\langle vV\rangle/R_L$, which is equal to the power dissipated by the capacitor through the resistance R_L, i.e., $\langle V^2\rangle/R_L$, see (3.6). Denoting steady state values by the subscript s, we have

$$\langle V^2\rangle_s = k_c\tau_p\langle vV\rangle_s. \tag{4.3}$$

The transducer's efficiency of converting mechanical to electrical power is given by the quotient of $k_c\tau_p\langle vV\rangle/R_L$ and $k_v\langle vV\rangle$, that is,

$$\eta_{\mathrm{me}} = \frac{k_c\tau_p\langle vV\rangle/R_L}{k_v\langle vV\rangle} = \frac{k_c\mathcal{C}}{k_v} < 1. \tag{4.4}$$

Our aim is to determine the overall efficiency of the conversion from the power supplied by the noise to the final net electrical power. This efficiency can be defined as the quotient of the corresponding powers,

$$\eta = \eta_{\mathrm{me}}\eta_{\mathrm{nm}} = \frac{\langle V^2\rangle_s/R_L}{\langle v\xi\rangle_s}, \tag{4.5}$$

where η_{me} is given in (4.4) and η_{nm} is the efficiency of power converted from the external noise to the power transferred from the oscillator to the transducer, that is, $\eta_{\mathrm{nm}} = k_v\langle vV\rangle_s/\langle v\xi\rangle_s$. By combining (3.1)–(3.6) in the steady state we have

$$\frac{\langle V^2\rangle_s}{R_L} = \phi\eta_{\mathrm{me}}\left(\langle v\xi\rangle_s + \frac{b}{k_c}\langle V\xi\rangle_s\right), \tag{4.6}$$

where ϕ is an electromechanical parameter defined as

$$\phi = \frac{k_c k_v \tau_p}{k_c\tau_p k_v(1+b\tau_p) + b + b\tau_p(b+\omega^2\tau_p)}. \tag{4.7}$$

Using (4.5), (4.6), (3.13), and (3.14), we can write the efficiency as

$$\eta = \frac{\phi}{R_L}\left[1 + \frac{b}{k_c}\frac{\langle V\xi\rangle_s}{\langle v\xi\rangle_s}\right] = \frac{\phi}{R_L}\left\{1 + b\frac{\int_0^\infty C(u)\left[\int_0^u G(u-z)e^{-z/\tau_p}dz\right]du}{\int_0^\infty G(u)C(u)du}\right\}. \tag{4.8}$$

Note that this is a general exact expression for the efficiency of energy harvesting by a linear electromechanical oscillator. Note further that the efficiency depends on the statistical properties of the noise only via the correlation function. The power dissipated by the oscillator can also be obtained in terms of the noise correlation function. In the steady state it follows from (4.2) and (4.6) that

$$b\langle v^2\rangle_s = -k_v\langle vV\rangle_s + \langle v\xi\rangle_s = (1-\phi)\int_0^\infty G(u)C(u)du - b\phi\int_0^\infty C(u)\left[\int_0^u G(u-z)e^{-z/\tau_p}dz\right]du, \tag{4.9}$$

where we have made use of (3.13) and (3.14). Let us analyze the limit case when $\tau_p \to \infty$. From Eqs. (2.4) and (2.5) it can be appreciate that the dynamical equations for the elecro-mechanical oscillator reduce to that of the mechanical oscillator $\ddot{x} + b\dot{x} + \omega_0^2 x = \xi(t)$. Since no power is converted to net electrical power this case lacks any practical interest. The efficiency of the conversion from the power supplied by the external noise to mechanical power is the quotient between the net power obtained and the power supplied by the noise, $\langle v\xi\rangle_s$. The net power of the mechanical oscillator is the difference between the power supplied by the noise and the dissipated power: $\langle v\xi\rangle_s - b\langle v^2\rangle_s$. In the stationary state it is expected that the mean energy is constant, so the net power is zero. This can also be checked from Eq. (4.2) taking $\tau_p \to \infty$. Therefore, in this limit both the net power and the efficiency are zero.

V. EXPONENTIALLY CORRELATED NOISES

In this section we apply our general results to an exponentially correlated stationary noise, i.e.,

$$C(u) = a\exp(-\lambda|u|), \tag{5.1}$$

where a and λ are the amplitude and the inverse of the correlation time $\lambda = \tau_c^{-1}$. The limit $\lambda \to \infty$ and $a \to \infty$, with $D = a/\lambda$ constant, corresponds to the white noise limit. It follows from (5.1), (3.13), and (3.14) that the power delivered by the noise is

$$\langle v\xi\rangle_s = \frac{a\lambda(1+\lambda\tau_p)}{\lambda(1+\lambda\tau_p)(b+\lambda)+\lambda k_c\tau_p k_v+\omega^2(1+\lambda\tau_p)}. \tag{5.2}$$

Equation (4.8) implies that the efficiency is given by

$$\eta = \phi\frac{1+\lambda\tau_p+b\tau_p}{1+\lambda\tau_p}. \tag{5.3}$$

Note that the efficiency of the whole conversion process, energy conversion from noise to net electrical energy, does not depend on the noise amplitude a but only on λ. The net electrical power is, from (4.6), (5.2), and (3.14), given by

$$P = \frac{\langle V^2\rangle_s}{R_L} = \phi\eta_{\rm me}\frac{a\lambda(1+\lambda\tau_p+b\tau_p)}{\lambda(1+\lambda\tau_p)(b+\lambda)+\lambda k_c\tau_p k_v+\omega^2(1+\lambda\tau_p)}. \tag{5.4}$$

The power dissipated by the oscillator,

$$b\langle v^2\rangle_s = a\lambda\frac{(1-\phi)(1+\lambda\tau_p)-b\phi\tau_p}{\lambda(1+\lambda\tau_p)(b+\lambda)+\lambda k_c\tau_p k_v+\omega^2(1+\lambda\tau_p)}, \tag{5.5}$$

is obtained from (4.9). We would like to reiterate that the efficiency and net electrical power depend on the noise source only via the noise correlation function $C(u)$. The above results hold, for example, for two such different noise sources as dichotomous Markov noise (non-Gaussian process) and Ornstein-Uhlenbeck (OU) noise (Gaussian process). Note also that the net electrical power is proportional the noise amplitude a; however its dependence on λ is more intricate. To check our analytical results we have performed numerical simulations of the Langevin equations (2.3) with an OU noise.

The numerical simulations are performed by solving numerically the Langevin equations (2.6c). We have discretized

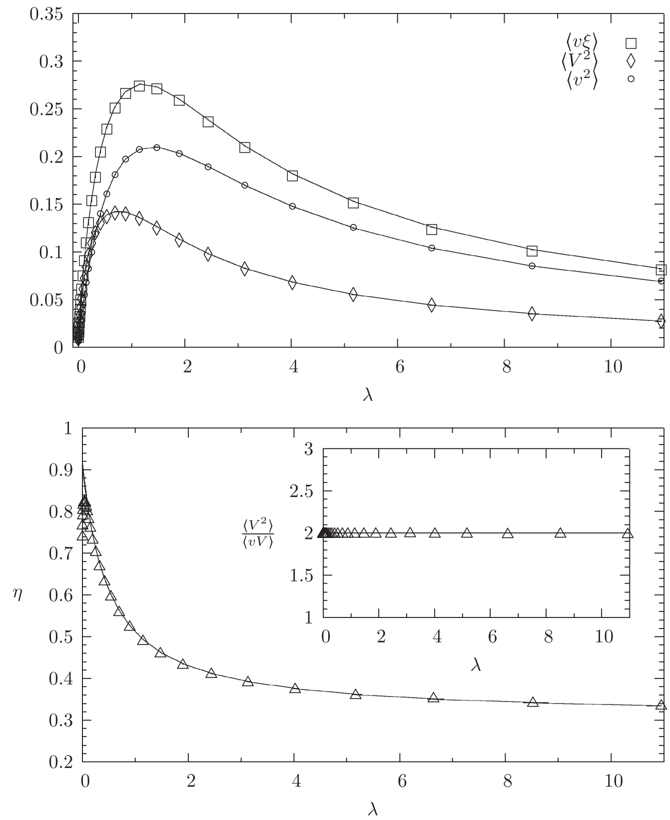

FIG. 2. Comparison of the results obtained from (5.2), (5.3), (5.4), and (5.5) (solid curves) with simulations (symbols). $a = b = \omega = k_c = k_v = V_0 = 1$; $\mathcal{C} = \tau_p = 2$. Inset: Comparison of the result obtained from (4.3) with numerical simulations.

the equations by using the so-called Heun algorithm [28]. At each time step we generate Gaussian white random numbers by means of the Box-Mueller-Wiener algorithm [28]. For the case of OU noise we have added to the system (2.6c) a fourth equation for the variable $\xi(t)$, namely, $d\xi/dt = -\xi\lambda + a\sqrt{\lambda}\eta(t)$, where $\eta(t)$ is Gaussian white noise with correlation $\langle\eta(t)\eta(t')\rangle = 2a^2\delta(t-t')/\lambda$. In this case the system of stochastic equations has been discretized again by using the Heun method and the random numbers for $\eta(t)$ have been generated by the Box-Mueller-Wiener algorithm.

We have averaged over 10^4 realizations of the noise for large times to determine $\langle v\xi\rangle_s$, P, and $\langle v^2\rangle_s$ independently of each other. In Fig. 2 we show the comparison between the results provided by (5.2), (5.3), (5.4), and (5.5) with numerical simulations. Note that the net electrical power P passes through a maximum, P^*, as λ increases. On the other hand, the efficiency decreases monotonically with λ. Consequently, the maximum efficiency does not occur when the net electrical power is maximal and vice versa. In the inset of the figure we have checked the result $\langle V^2\rangle_s = k_c\tau_p\langle vV\rangle_s$.

With the goal to optimize the performance of the energy conversion, we have studied how the maximum net electrical power, P^*, the efficiency at the maximum power, η^*, and the characteristic frequency of the noise at the maximum power, λ^*, depend on the parameters τ_p, k_c, k_v, and ω of the energy harvester. The three first parameters are related to the electrical

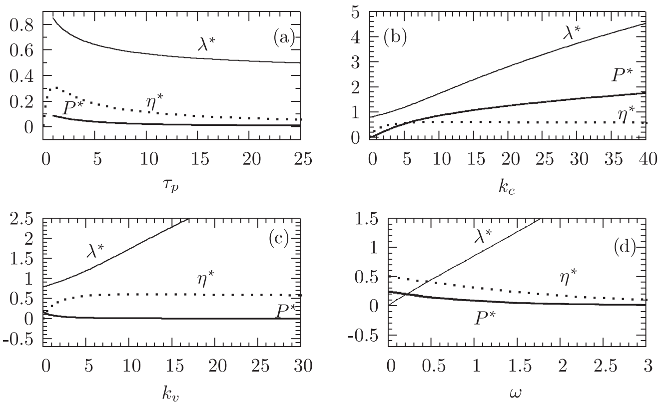

FIG. 3. Maximum net electrical power, P^*, efficiency at the maximum power, η^*, and characteristic frequency of the noise at the maximum power, λ^*, as a function of τ_p, k_c, k_v, and ω. In each panel we vary the corresponding parameter and set all other parameters equal to 1. For all plots we have taken $\mathcal{C} = 1$.

circuit of the transducer, and the last one characterizes the mechanical oscillator, the mass-spring system. In each panel of Fig. 3 we have varied the parameter specified and set the other parameters to 1. Both P^* and η^* decrease with ω [see Fig. 3(d). The maximum net electrical power, P^*, displays the same behavior as function τ_p, whereas η^* increases for small values of τ_p, passes through a maximum, and then decreases monotonically [see panel (a). The efficiency at maximum power η^* (see panels b and c) increases with k_c and k_v for small values, reaches a plateau for intermediate values, and then begins to decrease slowly (not shown)]. The roles of k_c and k_v on P^* are opposite to each other. P^* increases with k_c, but decreases with k_v [see Figs. 3(b) and 3(c)]. The characteristic frequency of the noise at the maximum power, λ^*, decreases monotonically as τ_p increases, whereas it increases monotonically as the remaining three parameters increase. In conclusion, to improve performance it would be desirable to tune τ_p and ω to low values and k_c and k_v to intermediate values.

To compare the above results with those for white noise, broadband random ambient vibrations, we define $a = D\lambda$ and take the limit $\lambda \to \infty$ with D being a nonzero constant in the above expressions. We find

$$\langle v\xi\rangle_s^W = D, \tag{5.6}$$

$$\eta^W = \phi, \tag{5.7}$$

$$\frac{\langle V^2\rangle_s^W}{R_L} = \frac{D\phi}{R_L}, \tag{5.8}$$

$$b\langle v^2\rangle_s^W = D(1-\phi), \tag{5.9}$$

where the label W denotes white noise. It is not difficult to check that

$$\langle v\xi\rangle_s^W > \langle v\xi\rangle_s^C, \tag{5.10}$$

$$\eta^W < \eta^C, \tag{5.11}$$

$$\frac{\langle V^2\rangle_s^W}{R_L} > \frac{\langle V^2\rangle_s^C}{R_L}, \tag{5.12}$$

$$b\langle v^2\rangle_s^W > b\langle v^2\rangle_s^C, \tag{5.13}$$

where the label C denotes colored noise, finite-bandwidth random ambient vibrations. In consequence, the white noise delivers higher power than the colored noise and the net electrical power obtained is also higher. However, in terms of efficiency, the electromechanical device is more efficient if the noise is colored than if it is white.

VI. CONCLUSIONS

We have derived exact analytical expressions for the power output and efficiency of converting ambient random vibrations into electrical power for a linear electromechanical oscillator driven by colored noise. An important finding is the fact that the efficiency and net electric power do depend only on the correlation function of the ambient noise. They do not depend on the probability distribution of the colored noise.

We evaluate the net electrical power and the efficiency explicitly for exponentially correlated noise. For such noises, the twin goals of maximum power and maximum efficiency cannot be achieved at the same time. A compromise has to be struck, and we find that the best performance of the energy harvester occurs if the natural frequency of the oscillator and the characteristic charging time of the capacitor are tuned to low values, while the parameters of the piezoelectric transducer are set to intermediate values. The efficiency of the overall conversion process depends only on the correlation time and the bandwidth of the noise and not on the noise amplitude. We also find that the device operates more efficiently in a finite-bandwidth environment, i.e., colored external noise, while the net electrical power is higher for broadband random vibrations, i.e., white noise.

While linear oscillators are widely used in energy harvesting, nonlinear oscillators have gained attention recently. We plan to investigate the performance of nonlinear energy harvesters driven by colored noise in future work.

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